Chapter 10  Hypothesis Tests Regarding a Parameter

Ch 10.1  The Language of Hypothesis Testing

Objective A : Set up a Hypothesis Testing

Hypothesis testing is a procedure, based on a sample evidence and probability, used to test statements regarding a characteristic of one or more populations.
- The null hypothesis \( H_0 \) is a statement to be tested.
- The alternate hypothesis \( H_1 \) is a statement that we are trying to find evidence to support.

Example 1: Set up \( H_0 \) and \( H_1 \).
(a) In the past, student average income was $6000 per year. An administrator believes the average income has increased.

(b) The percentage of passing a Math course was 50%. A math professor believes there is a decrease in the passing rate.

Objective B : Type I or Type II Error

Type I Error → Rejecting \( H_0 \) when \( H_0 \) is true.
Type II Error → Not rejecting \( H_0 \) when \( H_1 \) is true.

We use \( \alpha \) for the probability of making Type I error.
We use \( \beta \) for the probability of making Type II error.

For this statistics class, we only control the Type I error. \( (0.01 \leq \alpha \leq 0.10) \)

\( \alpha \) is also called the level of significance.

Objective C : State Conclusions to Hypothesis Tests

If \( H_0 \) is rejected, there is sufficient evidence to support the statement in \( H_1 \).
If \( H_0 \) is NOT rejected, there is NOT sufficient evidence to support the statement in \( H_1 \).
Example 1: In 2007, the mean SAT score on the reasoning test for all students was 710. A teacher believes that, due to the heavy use of multiple choice test questions, the mean SAT reasoning test has decreased.
(a) Determine $H_0$ and $H_1$.

(b) Explain what it would mean to make a Type I error.

(c) Explain what it would mean to make a Type II error.

(d) State the conclusion if the null hypothesis is rejected.

Example 2: The mean score on the SAT Math Reasoning exam is 516. A test preparation company states that the mean score of students who take its course is higher than 516.
(a) Determine the null and alternative hypotheses that would be used to test the effectiveness of the marketing campaign.

(b) If sample data indicate that the null hypothesis should not be rejected, state the conclusion of the company.

(c) Suppose, in fact, that the mean score of students taking the preparatory course is 522. Has a Type I or Type II error been made?

(d) If we wanted to decrease the probability of making a Type II error, would we need to increase or decrease the level of significance?
Example 3: According to the Centers for Disease Control, 15.2% of adults experience migraine headaches. Stress is a major contributor to the frequency and intensity of headaches. A massage therapist feels that she has a technique that can reduce the frequency and intensity of migraine headaches.
(a) Determine the null and alternative hypotheses that would be used to test the effectiveness of the massage therapist's techniques.

(b) A sample of 500 American adults who participated in the massage therapist's program results in data that indicate that the null hypothesis should be rejected. Provide a statement that supports the massage therapist's program.

(c) Suppose, in fact, that the percentage of patients in the program who experience migraine headaches is 15.2%. Was a Type I or Type II error committed?

Ch10.2 Hypothesis Tests for a Population Proportion

Objective A: Classical Approach

$Z$ – Test for a population proportion
A hypothesis test involving a population proportion can be considered as a binomial experiment.

The best point estimate of $p$, the population proportion, is a sample proportion, $\hat{p} = \frac{x}{n}$.

There are two methods for testing hypothesis.

Method 1: The Classical Approach

Hypothesis Testing Using the Classical Approach
If the sample proportion is too many standard deviations from the proportion stated in the null hypothesis, we reject the null hypothesis.

Method 2: The $P$-Value Approach

Hypothesis Testing Using the $P$-Value Approach
If the probability of getting a sample proportion as extreme or more extreme than the one obtained is small under the assumption the statement in the null hypothesis is true, reject the null hypothesis.
We will briefly introduce the classical approach.
The prefer method for this class is the $P$–value approach.

**Testing Hypotheses Regarding a Population Proportion, $p$.**

Use the following steps to perform a Proportion Z–Test provided that

- The sample is obtained by simple random sampling or the data result from a randomized experiment.
- $np_0(1 - p_0) \geq 10$.
- The sampled values are independent of each other.

**Step 1** Determine the null and alternative hypotheses. The hypotheses can be structured in one of three ways:

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<thead>
<tr>
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<th>Two-Tailed</th>
<th>Left-Tailed</th>
<th>Right-Tailed</th>
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</thead>
<tbody>
<tr>
<td>$H_0: p = p_0$</td>
<td>$H_0: p = p_0$</td>
<td>$H_0: p = p_0$</td>
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</tr>
<tr>
<td>$H_1: p \neq p_0$</td>
<td>$H_1: p &lt; p_0$</td>
<td>$H_1: p &gt; p_0$</td>
<td></td>
</tr>
</tbody>
</table>

*Note: $p_0$ is the assumed value of the population proportion.*

**Step 2** Select a level of significance $\alpha$, depending on the seriousness of making a Type I error.

**Classical Approach**

**Step 3** Compute the test statistic

$$z_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1 - p_0)}{n}}}$$

**Step 4** Compare the critical value with the test statistic.

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<tbody>
<tr>
<td>$z_0$ and $-z_0$</td>
<td>$-z_0$</td>
<td>$z_0$</td>
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</tr>
</tbody>
</table>

**Step 5** State the conclusion.
Objective A: Classical Approach

Z – Test for a population proportion

Example 1: Use the classical approach to test the following hypotheses.

\( H_0 : p = 0.25 \) versus \( H_1 : p < 0.25 \)

\( n = 400; \quad x = 96; \quad \alpha = 0.1 \)

(a) Setup \( H_0 \) and \( H_1 \)

(b) Use the sample data of \( n = 400 \) and \( x = 96 \) to compute the test statistic and round your answer to four decimal places.

(c) Use \( \alpha = 0.1 \) level of significance and StatCrunch to determine the critical value(s).

(d) Draw the \( Z \) – distribution that depicts the critical region and the \( Z \) – statistic.

(e) What conclusion can be drawn?
Hypothesis Testing about a Population Mean with unknown $\sigma$

**Objective A: Classical Approach**

$t$ - test for a population mean

### Testing Hypotheses Regarding a Population Mean

To test hypotheses regarding the population mean, we use the following steps, provided that

- The sample is obtained using simple random sampling or from a randomized experiment.
- The sample has no outliers, and the population from which the sample is drawn is normally distributed or the sample size, $n$, is large ($n \geq 30$).
- The sampled values are independent of each other.

**Step 1** Determine the null and alternative hypotheses. The hypotheses can be structured in one of three ways:

<table>
<thead>
<tr>
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<th>Two-Tailed</th>
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</thead>
<tbody>
<tr>
<td>$H_0$: $\mu = \mu_0$</td>
<td>$H_{\alpha}: \mu = \mu_0$</td>
<td>$H_{\alpha}: \mu &lt; \mu_0$</td>
<td>$H_{\alpha}: \mu &gt; \mu_0$</td>
</tr>
</tbody>
</table>

*Note: $\mu_0$ is the assumed value of the population mean.*

**Step 2** Select a level of significance, $\alpha$, depending on the seriousness of making a Type I error.

#### P-Value Approach

**By Hand Step 3** Compute the test statistic

$$ t_0 = \frac{\bar{x} - \mu_0}{s / \sqrt{n}} $$

which follows Student’s $t$-distribution with $n - 1$ degrees of freedom.

Use Table VI to approximate the $P$-value.

**Technology Step 3** Use a statistical spreadsheet or calculator with statistical capabilities to obtain the $P$-value. The directions for obtaining the $P$-value using the TI-83/84 Plus graphing calculator, MINITAB, Excel, and StatCrunch, are in the Technology Step-by-Step on page 508.

**Step 4** If the $P$-value < $\alpha$, reject the null hypothesis.

**Step 5** State the conclusion.

The $t$ - test procedure requires either that the sample was drawn from a normally distributed population or the sample size was greater than 30. Minor departures from normality will not adversely affect the results of the test. However, if the data include outliers, the $t$ - test procedure should not be used. A normality plot is used to test whether the sample was drawn from a normally distributed population. A boxplot is used to detect outliers.
Objective A: Classical Approach

t−test for a population mean

Example 1: Use StatCrunch to determine the critical value(s) for
(a) a right-tailed test for a population mean at the \( \alpha = 0.05 \) level of significance based on a sample size of \( n = 18 \).

\[
t_{\alpha} =
\]

(b) a left-tailed test for a population mean at the \( \alpha = 0.01 \) level of significance based on a sample size of \( n = 15 \).

\[
t_{\alpha} =
\]

(c) a two-tailed test for a population mean at \( \alpha = 0.05 \) level of significance based on a sample size of \( n = 25 \).

\[
t_{\alpha} =
\]

Example 2: Determine the \( t \)−statistic and the tail(s) of testing.
(a) \( H_0: \mu = 16 \) versus \( \mu > 16 \)
A random sample of size \( n = 10 \) was obtained from a population that was normally distributed produced a mean of 16.5 with a standard deviation of 0.4.

\[
t_0 = \frac{\overline{x} - \mu_0}{s / \sqrt{n}}
\]

(b) \( H_0: \mu = 72 \) versus \( \mu \neq 72 \)
A random sample of size \( n = 15 \) was obtained from a population that was normally distributed produced a mean of 73.5 with a standard deviation of 17.1.

\[
t_0 = \frac{\overline{x} - \mu_0}{s / \sqrt{n}}
\]
Example 3: Use the classical approach to test \( H_0: \mu = 0.3 \) versus \( \mu \neq 0.3 \), a random sample of size \( n = 16 \) is obtained from a population that is known to be normally distributed.

(a) Setup \( H_0 \) and \( H_1 \)

(b) If \( \bar{x} = 0.295 \), \( s = 0.168 \), compute the test statistic and round the answer to three decimal places.

(c) If the researcher decides to test this hypothesis at the \( \alpha = 0.05 \) level of significance, use StatCrunch to determine the critical value(s).

(d) Draw a \( t \)-distribution that depicts the critical region and the \( t \)-statistic.

(e) What conclusion can be drawn?
Ch10.2 Hypothesis Tests for a Population Proportion

Objective B: P – Value Approach

Z – Test for a population proportion

A hypothesis test involving a population proportion can be considered as a binomial experiment.

The best point estimate of \( p \), the population proportion, is a sample proportion, \( \hat{p} = \frac{x}{n} \), provided the sample is obtained by

1) simple random sampling;
2) \( np_0(1 - p_0) \geq 10 \) to guarantee that a normal distribution can be used to test hypothesis for \( H_0 : p = p_0 \);
3) the sampled values are independent of each other \( (n \leq 0.05N) \).

**Testing Hypotheses Regarding a Population Proportion, \( p \).**

Use the following steps to perform a \( Z – \) Test for a Proportion.

**Step 1** Determine the null and alternative hypotheses. The hypotheses can be structured in one of three ways:

<table>
<thead>
<tr>
<th>Hypothesis</th>
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Note: \( p_0 \) is the assumed value of the population proportion.

**Step 2** Select a level of significance \( \alpha \), depending on the seriousness of making a Type I error.

**P-Value Approach**

**By Hand**

**Step 3** Compute the test statistic

\[
Z_0 = \frac{\hat{p} - p_0}{\sqrt{p_0(1 - p_0)/n}}
\]

Use Table V to determine the \( P \)-value.

**Technology**

**Step 3** Use a statistical spreadsheet or calculator with statistical capabilities to obtain the \( P \)-value. The directions for obtaining the \( P \)-value using the TI-83/84 Plus graphing calculator, MINITAB, Excel, and StatCrunch are in the Technology Step-by-Step on page 496.

**Step 4** If \( P \)-value < \( \alpha \), reject the null hypothesis.

**Step 5** State the conclusion.
Example 1: The percentage of physicians who are women is 27.9%. In a survey of physicians employed by a large university health system, 45 of 120 randomly selected physicians were women. Use the \( P \)-value approach to determine whether there is sufficient evidence at the 0.05 level of significance to conclude that the proportion of women physicians at the university health system exceeds 27.9%?

(a) Setup

(b) Standardize the sample data

(c) Use StatCrunch to find the \( P \)-value.

(d) Draw the \( Z \)-distribution and shade the area that represents \( \alpha \) and the \( P \)-value respectively.

(e) What conclusion can be drawn?
Example 2: Redo Example --> Use StatCrunch to calculate part (b) and part (c) in one command.

(a) Setup

(b) Use StatCrunch to find $Z_0$ and its corresponding $P$-value.

(c) Conclusion

Example 3: In 2000, 58% of females aged 15 years of age and older lived alone, according to the U.S. Census Bureau. A sociologist tests whether this percentage is different today by conducting a random sample of 500 females aged 15 years of age and older and finds that 285 are living alone. Use the $P$-value approach and StatCrunch to determine whether there is sufficient evidence at the $\alpha = 0.10$ level of significance to conclude that the proportion has changed since 2000.
**P-Value Approach**

**Ch10.3 Hypothesis Testing about a Population Mean with unknown \( \sigma \)**

**Objective B : \( P \) – Value Approach**

\( t \) – test for a population mean

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**DEFINITION**

A **P-value** is the probability of observing a sample statistic as extreme or more extreme than the one observed under the assumption that the statement in the null hypothesis is true. Put another way, the **P-value** is the likelihood or probability that a sample will result in a statistic such as the one obtained if the null hypothesis is true.

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**Testing Hypotheses Regarding a Population Mean**

To test hypotheses regarding the population mean, we use the following steps, provided that

- The sample is obtained using simple random sampling or from a randomized experiment.
- The sample has no outliers, and the population from which the sample is drawn is normally distributed or the sample size, \( n \), is large \( (n \approx 30) \).
- The sampled values are independent of each other.

**Step 1** Determine the null and alternative hypotheses. The hypotheses can be structured in one of three ways:

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**Note:** \( \mu_0 \) is the assumed value of the population mean.

**Step 2** Select a level of significance, \( \alpha \), depending on the seriousness of making a Type I error.

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**P-Value Approach**

**By Hand Step 3** Compute the **test statistic**

\[
t_0 = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}
\]

which follows Student’s \( t \)-distribution with \( n - 1 \) degrees of freedom.

Use Table VI to approximate the **\( P \)-value**.

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</thead>
<tbody>
<tr>
<td>The sum of the area in the tails is the <strong>( P )-value</strong></td>
<td>The area left of ( t ) is the <strong>( P )-value</strong></td>
<td>The area right of ( t ) is the <strong>( P )-value</strong></td>
</tr>
<tr>
<td>( -t_0 )</td>
<td>( t_0 )</td>
<td>( t_0 )</td>
</tr>
</tbody>
</table>

**Technology Step 3** Use a statistical spreadsheet or calculator with statistical capabilities to obtain the **\( P \)-value**. The directions for obtaining the **\( P \)-value** using the TI-83/84 Plus graphing calculator, MINITAB, Excel, and StatCrunch, are in the Technology Step-by-Step on page 508.

**Step 4** If the **\( P \)-value** < \( \alpha \), reject the null hypothesis.

**Step 5** State the conclusion.
The $t$-test procedure requires either that the sample was drawn from a normally distributed population or the sample size was greater than 30. Minor departures from normality will not adversely affect the results of the test. However, if the data include outliers, the $t$-test procedure should not be used. A normality plot is used to test whether the sample was drawn from a normally distributed population. A boxplot is used to detect outliers.

**Example 1:**

(a) Draw a $t$-distribution with the area that represents the $P$-value shaded.

(b) Use StatCrunch to find the $P$-value for each test value, $t_0$.

(c) Use $\alpha = 0.05$ to determine whether to reject $H_0$ or not.

(i) $t_0 = 2.624$, $n = 15$, right-tailed

(ii) $t_0 = -2.321$, $n = 23$, left-tailed

(iii) $t_0 = 1.562$, $n = 17$, two-tailed
Example 3: A survey of 15 large U.S. cities finds that the average commute time one way is 25.4 minutes. A chamber of commerce executive feels that the commute in his city is less and wants to publicize this. He randomly selects 25 commuters and finds the average is 22.1 minutes with a standard deviation of 5.3 minutes. At \( \alpha = 0.01 \), is there enough evidence to support the claim? Use the \( P \)-value approach.

(a) Setup

(b) Use StatCrunch to find \( t_o \) and its corresponding \( P \)-value.

(c) Conclusion.

Example 4: Michael Sullivan, son of the author, decided to enroll in a reading course that allegedly increases reading speed and comprehension. Prior to enrolling in the class, Michael read 198 words per minute (wpm). The following data represent the words per minute read for 10 different passages read after the course.

<table>
<thead>
<tr>
<th>206</th>
<th>217</th>
<th>197</th>
<th>199</th>
<th>210</th>
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<tbody>
<tr>
<td>210</td>
<td>197</td>
<td>212</td>
<td>227</td>
<td>209</td>
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</tbody>
</table>
(a) Because the sample size is small, we must verify that reading speed is normally distributed and the sample does not contain any outliers. Perform a normal probability plot (QQ plot) and a boxplot. Are the conditions for testing a $t$-hypothesis satisfied?

(b) Was the class effective? Use the $\alpha = 0.10$ level of significance.

(i) Setup

(ii) Use StatCrunch to find $t_o$ and its corresponding $P$-value.

(iii) Conclusion.
P-Value Approach

Ch10  Hypothesis Tests for a Population Standard Deviation  (Supplemental Materials)

Objective B : P – Value Approach

$\chi^2$ – Test about a population variance or standard deviation

The concepts are similar to the P-value approach for Ch10.2 and Ch10.3 except a Chi-Square distribution is used. To test hypotheses about the population variance or standard deviation, two conditions must be met: 1) the sample is obtained using simple random sampling and 2) the population is normally distributed. Recall: distribution is not symmetric and the values of are non-negative.

Example 1: A machine fills bottles with 64 fluid ounces of liquid. The quality-control manager determines that the fill levels are normally distributed with a mean of 64 ounces and a standard deviation of 0.42 ounce. He has an engineer recalibrate the machine in an attempt to lower the standard deviation. After the recalibration, the quality-control control manager randomly selects 19 bottles from the line and determines that the standard deviation is 0.38 ounce. Is there less variability in the filling machine? Use the level of significant.

(a) Setup

(b) Use StatCrunch to find the $P$ – value.

(c) Draw the distribution and shade the area that represents and the value respectively.

(d) What conclusion can be drawn?
Example 2: Data obtained from the National Center for Health Statistics show that men between the ages of 20 to 29 have a mean height of 69.3 inches, with a standard deviation of 2.9 inches. A baseball analyst wonders whether the standard deviation of heights of major-league baseball is less than 2.9 inches. The heights (in inches) of 20 randomly selected players are shown below.

72  74  71  72  76  70  77  75  72  72  77  72  75  70  73  73  75  73  74  74

(a) Verify the data are normally distributed by drawing a normal probability plot.

(b) Use StatCrunch to compute the sample standard deviation.

(c) Test the notion at the $\alpha = 0.10$ level of significance.

Setup:

P-Value:

Conclusion: