## Parent Functions and Their Graphs

<table>
<thead>
<tr>
<th>Linear</th>
<th>Quadratic</th>
<th>Cubic</th>
<th>Absolute Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x) = x$</td>
<td>$f(x) = x^2$</td>
<td>$f(x) = x^3$</td>
<td>$f(x) =</td>
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<table>
<thead>
<tr>
<th>Radical</th>
<th>Rational</th>
<th>Greatest Integer</th>
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</thead>
<tbody>
<tr>
<td>$f(x) = \sqrt{x}$</td>
<td>$f(x) = \frac{1}{x}$</td>
<td>$f(x) = \lfloor x \rfloor$</td>
</tr>
</tbody>
</table>

- All pass through $(1,1)$.
- All but rational pass through $(0,0)$. 

\[ \text{All pass through } (1,1). \text{ All but rational pass through } (0,0). \]
Vertical Translations

Add to move up
Subtract to move down

Translated up 2 units: $h(x) = f(x) + 2$
$y = x^2 + 2$

Translated down 5 units: $g(x) = f(x) - 5$
$y = x^2 - 5$

Parent Graph: $f(x)$
**Horizontal Translations**

Add to move left
Subtract to move right

Translated left 2 units: \( h(x) = f(x+2) \)

Translated right 2 units: \( g(x) = f(x-2) \)

Parent Graph: \( f(x) \)

\[ y = x^2 \]
Reflected over the y-axis: $h(x) = f(-x)$

Reflected over the x-axis: $g(x) = -f(x)$

Parent Graph: $f(x)$

$y = \sqrt{x}$
Dilations

\[ c \cdot f(x) \]

\[ |c| < 1 \]
Graph narrows

\[ |c| > 1 \]
Graph widens

Narrows: \( g(x) = 2 \, f(x) \)

\[ y = 2x^2 \]

Widens: \( h(x) = 0.5 \, f(x) \)

\[ y = \frac{1}{2} x^2 \]

Parent Graph: \( f(x) \)
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<th>Rigid Transformations</th>
<th>Non-rigid Transformations</th>
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<td><strong>Function Notation</strong></td>
<td><strong>Description of transformation</strong></td>
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<tr>
<td>$f(x) = f(x) + c$</td>
<td>Translate up</td>
</tr>
<tr>
<td>$f(x) = f(x) - c$</td>
<td>Translate down</td>
</tr>
<tr>
<td>$f(x) = f(x + c)$</td>
<td>Translate left</td>
</tr>
<tr>
<td>$f(x) = f(x - c)$</td>
<td>Translate right</td>
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<tr>
<td>$f(x) = -f(x)$</td>
<td>Reflect over x-axis</td>
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<td>$f(x) = f(-x)$</td>
<td>Reflect over y-axis</td>
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<tr>
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<tr>
<td>$f(x) = c \cdot f(x)$</td>
<td>Narrows/Vertical Stretch</td>
</tr>
<tr>
<td>$f(x) = \frac{1}{c} \cdot f(x)$</td>
<td>Widens/Vertical Shrink</td>
</tr>
<tr>
<td>$f(x) = f(cx)$</td>
<td>Narrows/Horizontal Shrink</td>
</tr>
<tr>
<td>$f(x) = f\left(\frac{1}{c}\right)$</td>
<td>Widens/Horizontal Stretch</td>
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Rigid transformations hold their shape; non-rigid transformations change shape.
EXAMPLE 1:
Write the equation for the graph.

Parent graph: $f(x) = \frac{1}{x}$

moved down 3
moved right 4

$f(x-4) - 3$

$y = \frac{1}{x-4} - 3$
EXAMPLE 2:
Identify each transformation.
\[ f(x) = -\frac{1}{3}(x - 4)^3 + 2 \]
Reflected over the x-axis, vertical shrink, and translated up 2 and right 4 units.
EXAMPLE 3:
Write the equation of the transformed function given a description.

A quadratic function that has been reflected in the x-axis and translated left 2 units and up 9 units.

\[ f(x) = x^2 \]

\[ -(f(x+2))^2 + 9 \]

\[ y = -(x+2)^2 + 9 \]
EXAMPLE 4:
Use transformations to sketch the graph of the function given $f(x) = \sqrt{x}$

$2f(x + 2) - 3$

"slope" $\frac{1}{2}$ left down $\frac{3}{2}$
4. Quadratic opens down "slope" is \(-2\)

\[ y = -2x^2 \]
8. \[ g(x) = 6 - |x + 5| \]

\[ y = -|x + 5| + 6 \]

- Reflects over the x-axis
- Translated left 5
- Translated up 6
11. A radical function is only in the 2nd quadrant, and begins at (-2, 0).

\[ f(x) = \sqrt{x} \]

reflected in the y-axis left 2

\[ f(-x+2) \]

\[ y = \sqrt{-x+2} \]