Section 12.1  
First Derivative & Graphs

1) Theorem 1: Increasing & Decreasing Functions for the interval \((a, b)\).

| \(f'(x)\) | \(f(x)\) | Graph of \(f\) | Examples |

2) Critical values:
The values of \(x\) in the domain of \(f\) where \(f'(x) = 0\) or \(f'(x)\) doesn't exist are called the critical values.

Ex. a)

Ex.

Find (A) \(f'(x)\), (B) the critical values of \(f\), and (C) the partition points of \(f'\).

\(a) f(x) = x^3 - 27x + 30\).
b) \( f(x) = (x-9)^{2/3} \)

c) \( f(x) = \frac{5}{x-4} \)

**Theorem 2: Existence of Local Extrema**

If \( f \) is continuous on the interval \((a, b)\), \( c \) is the critical point in \((a, b)\) and \( f(c) \) is local extremum (maximum or minimum), then either \( f'(c) = 0 \) or \( f'(c) \) does not exist (is not defined).

Example: \( f'(c) = 0 \) horizontal tangent.
Ex. 2: Find the intervals on which \( f(x) \) is increasing, the intervals on which \( f(x) \) is decreasing, and the local extrema.

a) \( f(x) = -3x^2 - 12x \)

b) \( f(x) = -2x^3 + 3x^2 + 12x \)
c) \( f(x) = \frac{9}{x} + x \)

d) \( f(x) = \frac{3 - \frac{4}{x}}{x^2} \)

e) \( f(x) = \frac{x^2}{x+1} \)

f) \( f(x) = \sqrt[3]{(x-5)^2} \)
Ex. 3: Profit analysis: The graph of the total profit $P(x)$ (in $\$) from the sale of $x$ cordless electric screwdrivers is shown.

(A) Write a brief description of the graph of the marginal profit function $\frac{d}{dx}P(x)$, including a discussion of any x-intercepts.

(b) Sketch a possible graph of $y = P(x)$.
2.4 Average cost: A manufacturer incurs the following costs in producing x water ski vests in one day, for 0 < x < 150: fixed cost, $320; unit production cost, $20 per vest; equipment maintenance and repairs, 0.05x² $ So the cost of manufacturing x vests in one day is given by

\[ C(x) = 0.05x^2 + 20x + 320 \; 0 < x < 150 \]

a) What is the average cost \( \overline{C(x)} \) per vest if \( x \) vests are produced in one day?

b) Find the critical value \( C'(x) \), the intervals on which the average cost per vest is \( \downarrow \) and the interval on which the \( C(x) \) is \( \uparrow \) and local extrema
Ex. A drug is injected into the bloodstream of a patient through the right arm. The drug concentration in the bloodstream of the left arm at hour $t$ after the injection is $z$ by 
$$z(t) = \frac{0.28t}{t+4}$$
for $0 < t < 24$.

Find the CV, the intervals on which the drug concentration is $\downarrow$ and $\uparrow$ and the local extrema.