SOLUTIONS

MATH 238 **CHAPTER 14B WORKSHOP** **SPRING 2014**

1. Find the future value, at 3.75% interest, compounded continuously for 5 years, of the continuous income stream with rate of flow $f(t) = 3000e^{0.05t}$ $T = 5$ $r = 0.0375$

$$FV = e^{rT}\int_0^T f(t)e^{-rt}\,dt$$

$$= e^{0.0375(5)}\int_0^5 3000e^{0.05t}e^{-0.0375t}\,dt$$

$$= 3000e^{0.1875}\int_0^5 e^{0.0125t}\,dt =$$

$$\frac{3000}{0.0125}e^{0.1875}\left(e^{0.0125(5)} - e^0\right) = \boxed{\$18{,}671}$$

2. Find the ~~consumers'~~ producers' surplus at a price level of $\bar{p} = \cancel{\$160}\ 98$ for the price-demand equation $p = S(x) = 10 + 0.1x + 0.0003x^2$ Find $\bar{x}$

$\bar{p} = 10 + 0.1\bar{x} + 0.0003\bar{x}^2$

$98 = 10 + 0.1\bar{x} + 0.0003\bar{x}^2$

$0.0003\bar{x}^2 + 0.1\bar{x} - 88 = 0$

use quadratic eq. $a = 0.0003$ $b = 0.1$ $c = -88$

$\Rightarrow$ you will get $\bar{x} = 400$

$$PS = \int_0^{\bar{x}}[\bar{p} - S(x)]\,dx$$

$$= \int_0^{400}[88 - 0.1x - 0.0003x^2]\,dx$$

$$= 88x - 0.05x^2 - 0.0001x^3\Big|_0^{400}$$

$$= \boxed{\$20{,}800}$$

3. Find the ~~producers'~~ consumers' surplus at a price level of $\bar{p} = \cancel{\$98}\ \$160$ for the price-supply equation $p = D(x) = 460 - 0.06x$

Find $\bar{x}$

$\bar{p} = 460 - 0.06\bar{x}$

$160 = 460 - 0.06\bar{x}$

$\bar{x} = 5000$

$$CS = \int_0^{\bar{x}}[D(x) - \bar{p}]\,dx = \int_0^{5000}[460 - 0.06x - 160]\,dx$$

$$= \int_0^{5000}(300 - 0.06x)\,dx = 300x - 0.03x^2\Big|_0^{5000}$$

$$= 300(5000) - 0.03(5000)^2 - 0 = \boxed{\$750{,}000}$$

(4-13)

For ~~11~~-~~20~~, integrate:

4. $\int xe^{5x}dx$

$u = x \quad dv = e^{5x}$

$du = dx \quad v = \frac{1}{5}e^{5x}$

$= x\left(\frac{1}{5}e^{5x}\right) - \int \frac{1}{5}e^{5x}dx$

$= \frac{x}{5}e^{5x} - \frac{1}{5}\cdot\frac{1}{5}e^{5x} + C = \boxed{\frac{x}{5}e^{5x} - \frac{1}{25}e^{5x} + C}$

5. $\int x^4 \ln x dx$

$u = \ln x \quad dv = x^4 dx$

$du = \frac{1}{x}dx \quad v = \frac{x^5}{5}$

$= \frac{x^5}{5}\ln x - \int \frac{x^5}{5}\cdot\frac{1}{x}dx$

$= \frac{x^5}{5}\ln x - \frac{1}{5}\int x^4 dx = \boxed{\frac{x^5}{5}\ln x - \frac{1}{25}x^5 + C}$

6. $\int_2^6 \ln 5x\, dx$

$u = \ln 5x \quad dv = dx$

$du = \frac{1}{5x}\cdot 5dx \quad v = x$

$du = \frac{1}{x}dx$

$= x\ln 5x - \int \cancel{x}\cdot\frac{1}{\cancel{x}}dx$

$= x\ln 5x - x\Big|_2^6 = \boxed{11.802}$

$x^5 = x^3 \cdot x^2$

7. $\int 3x^5 e^{x^3} dx \longrightarrow \int e^u du$

$u = x^3 \quad dv = 3x^2 e^{x^3} dx$

$du = 3x^2 dx \quad v = e^{x^3}$

$= x^3 e^{x^3} - \int e^{x^3}(3x^2)dx \longrightarrow$ u substitution $u = x^3$

$du = 3x^2 dx$

$= x^3 e^{x^3} - \int e^u du = x^3 e^{x^3} - e^u + C$

$= \boxed{x^3 e^{x^3} - e^{x^3} + C}$

8. $\int \frac{e^x + 5}{e^x + 5x}dx \longrightarrow$ use substitution $u = e^x + 5x$

$du = (e^x + 5)dx$

$= \int \frac{1}{u}du = \ln|u| + C$

$= \ln|e^x + 5x| + C$

u substitution $u = -2x^2$

$du = -4x\,dx$

9. $\frac{-4}{-4}\int_0^2 xe^{-2x^2}dx$

$-\frac{1}{4}\int_0^2 e^{-2x^2}(-4x\,dx) = -\frac{1}{4}\int_0^{-8} e^u du = -\frac{1}{4}e^u\Big|_0^{-8}$

$= -\frac{1}{4}\left[e^{-8} - e^0\right] = -\frac{1}{4}\left[\frac{1}{e^8} - 1\right] = \boxed{0.2499}$

(7a) $\int 3x^2 e^{x^3} dx = \int e^u du \qquad u = x^3 \quad du = 3x^2 dx$

$= \boxed{e^{x^3} + C}$

by parts

10. $\int (x+7)^8 (x-3)\,dx$

$u = x-3 \quad dv = (x+7)^8\,dx$

$du = dx \quad v = \frac{1}{9}(x+7)^9$

$= (x-3)\left(\frac{1}{9}(x+7)^9\right) - \int \frac{1}{9}(x+7)^9\,dx$

$= \frac{1}{9}(x-3)(x+7)^9 - \frac{1}{9}\int (x+7)^9\,dx = \frac{1}{9}(x-3)(x+7)^9 - \frac{1}{9}\cdot\frac{1}{10}(x+7)^{10} + C$

$= \boxed{\frac{1}{9}(x-3)(x+7)^9 - \frac{1}{90}(x+7)^{10} + C}$

11. $\int \frac{\ln x}{\sqrt{x}}\,dx$

$u = \ln x \quad dv = \frac{1}{\sqrt{x}}\,dx = x^{-\frac{1}{2}}dx$

$du = \frac{1}{x}\,dx \quad v = \frac{x^{\frac{1}{2}}}{\frac{1}{2}} = 2x^{\frac{1}{2}}$

$= 2x^{\frac{1}{2}}\ln x - \int 2x^{\frac{1}{2}}\cdot\frac{1}{x}\,dx$

$= 2x^{\frac{1}{2}}\ln x - 2\int x^{-1/2}\,dx$

$= 2x^{\frac{1}{2}}\ln x - 2\frac{x^{1/2}}{\frac{1}{2}} + C = \boxed{2x^{\frac{1}{2}}\ln x - 4x^{\frac{1}{2}} + C}$

12. $\int x(\ln x)^3\,dx$

$u = (\ln x)^3 \quad dv = x\,dx$

$du = 3(\ln x)^2\cdot\frac{1}{x} \quad v = \frac{x^2}{2}$

$\frac{x^2}{2}(\ln x)^3 - \int 3\frac{x^2}{2}\cdot(\ln x)^2\cdot\frac{1}{x}dx$

$u = (\ln x)^2 \quad dv = x\,dx$

$\frac{x^2}{2}(\ln x)^3 - \frac{3}{2}\int x(\ln x)^2 dx$

$du = 2\ln x\,\frac{1}{x}dx \quad v = \frac{x^2}{2}$

$\frac{x^2}{2}(\ln x)^3 - \frac{3}{2}\left[\frac{x^2}{2}(\ln x)^2 - \underbrace{\int \frac{x^2}{2}\cdot 2\ln x\cdot\frac{1}{x}\,dx}_{\int x\ln x\,dx}\right] = \frac{x^2}{2}(\ln x)^3 - \frac{3}{4}x^2(\ln x)^2 + \frac{3}{2}\left[\frac{x^2}{2}\ln x - \int \frac{x^2}{2}\cdot\frac{1}{x}dx\right]$

$u = \ln x \quad du = \frac{1}{x}dx \quad dv = x\,dx$

$v = \frac{x^2}{2} \quad \frac{x^2}{2}\cdot\frac{1}{x}dx$

13. $\int_1^e \ln(x^4)\,dx$

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$= \boxed{\frac{x^2}{2}(\ln x)^3 - \frac{3}{4}x^2(\ln x)^2 + \frac{3}{4}x^2\ln x - \frac{3}{8}x^2 + C}$

$\frac{1}{2}\int x\,dx = \frac{1}{2}\frac{x^2}{2} = \frac{1}{4}x^2$

Use Table II to find the integral:

integrals involving $\sqrt{u^2 - a^2}$

Formula 44

14. $\int \frac{x^2}{\sqrt{25x^2-36}}\,dx$

$u^2 = 25x^2 \quad a^2 = 36$

$u = 5x \quad a = 6$

$= \frac{1}{5}\int \frac{\frac{1}{25}u^2}{\sqrt{u^2-6^2}}\,du = \frac{1}{125}\int \frac{u^2}{\sqrt{u^2-6^2}}\,du$

$= \frac{1}{125}\left(\frac{1}{2}5x\sqrt{25x^2-36} + 36\ln\left|5x+\sqrt{25x^2-36}\right|\right) + C$

Formula 9 $a=7$ $b=3$

15. $\int_1^3 \frac{1}{x(7+3x)}dx = \int_{x=1}^{x=3} \frac{1}{u(a+bu)}du = \frac{1}{7}\ln\left|\frac{x}{7+3x}\right|\Big]_1^3$

$= 0.0898$

Formula 35

16. $\int \frac{\sqrt{9x^2+25}}{9x^2}dx$

$= \frac{1}{3}\left[-\frac{\sqrt{9x^2+25}}{3x} + \ln\left|3x+\sqrt{9x^2+25}\right|\right] + C$

Formula 20 $a=3$ $b=5$ $c=4$ $d=7$

$\int_0^1 \frac{3+5x}{4+7x}dx = \frac{5x}{7} + \frac{3(7)-5(4)}{7^2}\ln|4+7x|\Big|_0^1$

$= \frac{5x}{7} + \frac{1}{49}\ln|4+7x|\Big]_0^1 \approx \boxed{0.7915}$

17. $\int_0^1 \frac{3+5x}{4+7x}dx$

#13 $\int_1^e \ln(x^4)dx$

$u=\ln(x^4)$ $dv=dx$

$du=\frac{1}{x^4}(4x^3)dx$ $v=x$

$= x\ln(x^4) - \int_1^e \cancel{x}\cdot\frac{4}{\cancel{x}}dx$

$du = \frac{4}{x}dx$

$= x\ln x^4 - 4x\Big|_1^e$

$= e\ln e^4 - 4e - (\cancel{1\ln 1}^{0} - 4)$

$= e\cancel{(4)} - \cancel{4e} + 4$ $= \boxed{4}$