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1. (5pts) Find the particular solution for the differential equation  $\frac{dy}{dx} = \frac{2}{1+x}$ ;  $y(0) = 5$

$$dy = \int \frac{2}{1+x} dx = 2 \int \frac{1}{1+x} dx \quad \begin{matrix} u = 1+x \\ du = dx \end{matrix}$$

$$= 2 \int \frac{1}{u} du = 2 \ln|u| + C = 2 \ln|1+x| + C$$

$$y(0) = 2 \ln(1) + C = 5 \rightarrow C = 5$$

$$y = 2 \ln|1+x| + 5$$



2. (5pts) Find the indefinite integral  $\int \left( 3\sqrt{x} + \frac{2}{\sqrt{x}} \right) dx = 3 \int x^{1/2} dx + 2 \int x^{-1/2} dx$

$$y = 3 \frac{x^{3/2}}{3/2} + 2 \frac{x^{1/2}}{1/2} + C$$

$$y = 2x^{3/2} + 4\sqrt{x} + C$$



3. (5pts) Find the indefinite integral  $\int \frac{\frac{8}{9} x^3}{\sqrt{2x^4+3}} dx$

$$\text{let } u = 2x^4 + 3 \\ du = 8x^3 dx$$

$$\int \frac{x^3}{\sqrt{2x^4+3}} dx = \frac{1}{8} \int \frac{1}{u^{1/2}} du = \frac{1}{8} \frac{u^{1/2}}{1/2} + C$$

$$= \frac{1}{4} \sqrt{2x^4+3} + C$$



4. (10 pts) Evaluate the integrals:

$$\begin{aligned} \text{a) } & \int_2^3 12(x^2-4)^5 x dx && u = x^2 - 4 \\ & && du = 2x dx \\ & = 6 \int_2^3 2x(x^2-4)^5 dx = 6 \int_0^5 u^5 du = 6 \cdot \frac{u^6}{6} \Big|_0^5 = u^6 \Big|_0^5 \\ & = 5^6 - 0^6 = 15,625 \end{aligned}$$



$$\begin{aligned} \text{b) } & \int_1^e \frac{\ln t}{t} dt && u = \ln t \\ & && du = \frac{1}{t} dt \\ & = \int_0^1 u du = \frac{u^2}{2} \Big|_0^1 = \frac{1}{2} \end{aligned}$$

