Math 238 Ch.10 Additional Problems

10.4 #66

$S(t) = 2\sqrt{t+6}$

(A) $S(t+h) = 2\sqrt{t+h+6}$

$S(t+h) - S(t) = 2(\sqrt{t+h+6} - \sqrt{t+6})$

$\dfrac{S(t+h)-S(t)}{h} = \dfrac{2(\sqrt{t+h+6} - \sqrt{t+6})}{h}$

$$S'(t) = \lim_{h\to 0} \frac{2(\sqrt{t+h+6} - \sqrt{t+6})}{h} \cdot \left(\frac{\sqrt{t+h+6} + \sqrt{t+6}}{\sqrt{t+h+6} + \sqrt{t+6}}\right)$$

$$= \lim_{h\to 0} \frac{2[\cancel{t}+h+\cancel{6}-\cancel{t}-\cancel{6}]}{h(\sqrt{t+h+6} + \sqrt{t+6})}$$

$$= \lim_{h\to 0} \frac{2\cancel{h}}{\cancel{h}(\sqrt{t+h+6} + \sqrt{t+6})} = \frac{2}{\sqrt{t+6} + \sqrt{t+6}} = \frac{\overset{1}{\cancel{2}}}{\cancel{2}\sqrt{t+6}}$$

$$\boxed{S'(t) = \frac{1}{\sqrt{t+6}}}$$

(B) $S'(10) = \dfrac{1}{\sqrt{10+6}} = \dfrac{1}{\sqrt{16}} = \dfrac{1}{4} = 0.25$

$S(t) = 2\sqrt{10+6} = 2\sqrt{16} = 2(4) = 8$

After 10 months, the sales are 8 million dollars and are increasing at a rate of 0.25 million.

(C) After 11 months, sales are $8 + 0.25 = 8.25$
\$8.25 million

After 12 months, sales are $8.25 + 0.25 = 8.50$
\$8.50 million

10.5

#51 P.533

$f(x) = 3x^4 - 6x^2 - 7$

(A) $f'(x) = 12x^3 - 12x$

(B) $f'(2) = 12(2)^3 - 12(2) = 72$

(C) at $x = 2$, $(2, 17)$

$f(2) = 3(2^4) - 6(2^2) - 7 = 17$

equation of the tangent line at $x=2$

$y - 17 = 72(x-2)$

$y - 17 = 72x - 144$

$\boxed{y = 72x - 127}$

(D) the tangent line is horizontal when the slope $= 0$ $\Rightarrow f'(x) = 0$

Factor

$\Rightarrow 12x^3 - 12x = 0$

$12x(x^2 - 1) = 0$

$12x(x-1)(x+1) = 0$

$\boxed{x = 0 \quad x = -1 \quad x = 1}$

tangent line is horizontal when $x=0$, $x = -1$ and $x = 1$

#53 $f(x) = 176x - 16x^2$

(A) $v(x) = f'(x) = 176 - 32x$

(B) $x = 0 \Rightarrow v(0) = 176$

$x = 3\text{sec} \Rightarrow v(3) = 176 - 32(3) = 80$

(C) time when $v = 0 \Rightarrow 176 - 32x = 0$

$\Rightarrow x = \frac{176}{32} = 5.5$

velocity is 0 when $x = 5.5$ seconds

10.6

#39 $\overline{C} = 400x^{-1} + 5 + \frac{1}{2}x \quad x \geq 1$

production from 20/hr to 25/hr

$dx = 25 - 20 = 5 \qquad x = 20$

$d\overline{C} = \overline{C}'\,dx$

$= (-400x^{-2} + \frac{1}{2})dx$

$= (-\frac{400}{x^2} + \frac{1}{2})\,dx$

$= (-\frac{400}{20^2} + \frac{1}{2}) \cdot 5$

$= (-\frac{1}{2}) \cdot 5 = -\frac{5}{2}$

$d\overline{C} = -2.5$

$\boxed{-\$2.50}$

\$ 2.50 decrease

10.7 #33

$P(x) = 30x - 0.03x^2 - 750 \qquad 0 \leq x \leq 1{,}000$

(A) Find the ave. profit per mower if 50 mowers are produced

$\overline{P}(x) = \frac{30x - 0.03x^2 - 750}{x} = 30 - 0.03x - 750x^{-1}$

$\overline{P}(50) = 30 - 0.03(50) - \frac{750}{50} = 13.5$

$\boxed{\$13.50}$

(B) Find the marginal profit at a production level of 50 mowers and interpret the results

$\overline{P}'(x) = -0.03 + 750x^{-2} = 0.03 + \frac{750}{x^2}$

$P'(50) = -0.03 + \frac{750}{50^2} = 0.27$

$\boxed{\$0.27}$

At a production level of 50 mowers, the average profit per mower is increasing at a rate of \$0.25 per mower

(C) $\overline{P}(50) + 0.27 = 13.50 + 0.27 = 13.77$

$\boxed{\approx \$13.77}$

Review problems done in class and in the review sessions

Good Luck!