Chapter 2

Integers and Introduction to Integers
2.1

Introduction to Integers
Numbers greater than 0 are called **positive numbers**. Numbers less than 0 are called **negative numbers**.
Some signed numbers are integers.

The integers are
\{ \ldots, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, \ldots \}
−3 indicates “negative three.”
3 and +3 both indicate “positive three.”
The number 0 is neither positive nor negative.
We compare integers just as we compare whole numbers. For any two numbers graphed on a number line, the number to the right is the greater number and the number to the left is the smaller number.

< means “is less than”

> means “is greater than”
The graph of $-5$ is to the left of $-3$, so $-5$ is less than $-3$, written as $-5 < -3$.

We can also write $-3 > -5$.

Since $-3$ is to the right of $-5$, $-3$ is greater than $-5$. 
The **absolute value** of a number is the number’s distance from 0 on the number line. The symbol for absolute value is | |.

|2| is 2 because 2 is 2 units from 0.

|−2| is 2 because −2 is 2 units from 0.
Since the absolute value of a number is that number’s *distance* from 0, the absolute value of a number is always 0 or positive. It is never negative.

\[
|0| = 0 \quad |−6| = 6
\]

zero \hspace{1cm} \text{a positive number}
Two numbers that are the same distance from 0 on the number line but are on the opposite sides of 0 are called **opposites**.

5 and \(-5\) are opposites.
Opposite Numbers

5 is the opposite of \(-5\) and \(-5\) is the opposite of 5.

The opposite of 4 is \(-4\) is written as

\[-(4) = -4\]

The opposite of \(-4\) is 4 is written as

\[-(-4) = 4\]

\[-(-4) = 4\]

If \(a\) is a number, then \(-(-a) = a\).
Remember that 0 is neither positive nor negative. Therefore, the opposite of 0 is 0.
2.2

Adding Integers
Adding Two Numbers with the Same Sign

\[ 2 + 3 = 5 \]

Start: 2 3
End:

-6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6

\[ -2 + (-3) = -5 \]

Start: -2 -3
End:

-6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6
Adding Two Numbers with the Same Sign

Step 1: Add their absolute values.

Step 2: Use their common sign as the sign of the sum.

Examples: $-3 + (-5) = -8$

$5 + 2 = 7$
Adding Two Numbers with Different Signs

2 + (−3) = −1

Start

End

−6 −5 −4 −3 −2 −1 0 1 2 3 4 5 6

−2 + 3 = 1
Adding Two Numbers with Different Signs

Step 1: Find the larger absolute value minus the smaller absolute value.

Step 2: Use the sign of the number with the larger absolute value as the sign of the sum.

Examples: $-4 + 5 = 1 \quad 6 + (-8) = -2$
Helpful Hint

If $a$ is a number, then

$-a$ is its opposite.

\[
a + (-a) = 0
\]
\[
-a + a = 0
\]

The sum of a number and its opposite is 0.
Helpful Hint

Don’t forget that addition is commutative and associative. In other words, numbers may be added in any order.
Evaluating Algebraic Expressions

Evaluate \( x + y \) for \( x = 5 \) and \( y = -9 \).

Replace \( x \) with 5 and \( y \) with \(-9\) in \( x + y \).

\[
\begin{align*}
x + y &= (5) + (-9) \\
&= -4
\end{align*}
\]
2.3

Subtracting Integers
Subtracting Integers

To subtract integers, rewrite the subtraction problem as an addition problem. Study the examples below.

\[9 - 5 = 4\]

\[9 + (-5) = 4\]

Since both expressions equal 4, we can say

\[9 - 5 = 9 + (-5) = 4\]

Subtracting Two Numbers

If \( a \) and \( b \) are numbers, then

\[
a - b = a + (-b).
\]

To subtract two numbers, add the first number to the opposite (called additive inverse) of the second number.
### Subtracting Two Numbers

Subtraction can be thought of as adding the opposite of a number.

<table>
<thead>
<tr>
<th>Subtraction</th>
<th>First Number</th>
<th>Opposite of Second Number</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>$7 - 4$</td>
<td>$7$</td>
<td>$-4$</td>
<td>$3$</td>
</tr>
<tr>
<td>$-5 - 3$</td>
<td>$-5$</td>
<td>$3$</td>
<td>$-8$</td>
</tr>
<tr>
<td>$3 - (-6)$</td>
<td>$3$</td>
<td>$6$</td>
<td>$9$</td>
</tr>
<tr>
<td>$-8 - (-2)$</td>
<td>$-8$</td>
<td>$2$</td>
<td>$-6$</td>
</tr>
</tbody>
</table>

Adding and Subtracting Integers

If a problem involves adding or subtracting more than two integers, rewrite differences as sums and add. By applying the associative and commutative properties, add the numbers in any order.

\[ 9 - 3 + (-5) - (-7) = 9 + (-3) + (-5) + 7 \]

\[ \underbrace{6 + (-5) + 7}_{1 + 7} \]

\[ \underbrace{1 + 7}_{8} \]
Evaluate $x - y$ for $x = -6$ and $y = 8$.

Replace $x$ with $-6$ and $y$ with $8$ in $x - y$.

\[ x - y = \begin{array}{c} (-6) - (8) \\ = (-6) + (-8) \\ = -14 \end{array} \]
2.4

Multiplying and Dividing Integers
Consider the following pattern of products.

First factor
decreases by 1
each time.

<table>
<thead>
<tr>
<th>First factor</th>
<th>Product decreases by 5 each time.</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 • 5 = 15</td>
<td></td>
</tr>
<tr>
<td>2 • 5 = 10</td>
<td></td>
</tr>
<tr>
<td>1 • 5 = 5</td>
<td></td>
</tr>
<tr>
<td>0 • 5 = 0</td>
<td></td>
</tr>
</tbody>
</table>

This pattern continues as follows.

-1 • 5 = -5
-2 • 5 = -10
-3 • 5 = -15

This suggests that the product of a negative number and a positive number is a negative number.
Observe the following pattern.

\[ 2 \cdot (-5) = -10 \]
\[ 1 \cdot (-5) = -5 \]
\[ 0 \cdot (-5) = 0 \]

This pattern continues as follows.

\[ -1 \cdot (-5) = 5 \]
\[ -2 \cdot (-5) = 10 \]
\[ -3 \cdot (-5) = 15 \]

This suggests that the product of two negative numbers is a positive number.
Multiplying Integers

The product of two numbers having the same sign is a positive number.

\[ 2 \cdot 4 = 8 \quad \text{and} \quad -2 \cdot (-4) = 8 \]

The product of two numbers having different signs is a negative number.

\[ 2 \cdot (-4) = -8 \quad \text{and} \quad -2 \cdot 4 = -8 \]
Multiplying Integers

Product of Like Signs

\(( + )( + ) = +\)  \((−)(−) = +\)

Product of Different Signs

\((−)( + ) = −\)  \(( + )(−) = −\)
If we let (–) represent a negative number and (+) represent a positive number, then

The product of an even number of negative numbers is a positive result.

\[
(\neg)(\neg) = (+)
\]

\[
(\neg)(\neg)(\neg) = (\neg)
\]

\[
(\neg)(\neg)(\neg)(\neg) = (+)
\]

The product of an odd number of negative numbers is a negative result.

\[
(\neg)(\neg)(\neg)(\neg)(\neg) = (\neg)
\]
Division of integers is related to multiplication of integers.

\[
\frac{6}{2} = 3 \quad \text{because} \quad 3 \cdot 2 = 6
\]

\[
\frac{-6}{2} = -3 \quad \text{because} \quad -3 \cdot 2 = -6
\]

\[
\frac{6}{-2} = -3 \quad \text{because} \quad -3 \cdot (-2) = 6
\]

\[
\frac{-6}{-2} = 3 \quad \text{because} \quad 3 \cdot (-2) = -6
\]
Dividing Integers

The quotient of two numbers having the same sign is a positive number.

\[ 12 \div 4 = 3 \qquad -12 \div (-4) = 3 \]

The quotient of two numbers having different signs is a negative number.

\[ -12 \div 4 = -3 \qquad 12 \div (-4) = -3 \]
Dividing Numbers

Quotient of Like Signs

\[
\frac{(+)}{(+) = +} \quad \frac{(-)}{(-)} = +
\]

Quotient of Different Signs

\[
\frac{(+)}{(-)} = - \quad \frac{(-)}{(+)} = -
\]
2.5

Order of Operations
Order of Operations

1. Perform all operations within parentheses ( ), brackets [ ], or other grouping symbols such as fraction bars, starting with the innermost set.

2. Evaluate any expressions with exponents.

3. Multiply or divide in order from left to right.

4. Add or subtract in order from left to right.
Simplify $4(5 - 2) + 4^2$.

\[
4(5 - 2) + 4^2 = 4(3) + 4^2
\]

Simplify inside parentheses.

\[
= 4(3) + 16
\]

Write $4^2$ as 16.

\[
= 12 + 16
\]

Multiply.

\[
= 28
\]

Add.
When simplifying expressions with exponents, parentheses make an important difference.

\((-5)^2\) and \(-5^2\) do not mean the same thing.

\((-5)^2\) means \((-5)(-5) = 25\).

\(-5^2\) means the opposite of \(5 \cdot 5\), or \(-25\).

Only with parentheses is the \(-5\) squared.
2.6

Solving Equations: The Addition and Multiplication Properties
Equations

Statements like $5 + 2 = 7$ are called **equations**.

An equation is of the form **expression = expression**.

An equation can be labeled as

$$x + 5 = 9$$

**Equal sign**

**left side**

**right side**
When an equation contains a variable, deciding which values of the variable make an equation a true statement is called **solving** an equation for the variable.

A **solution** of an equation is a value for the variable that makes an equation a true statement.
Solutions of Equations

Determine whether a number is a solution:

Is $-2$ a solution of the equation $2y + 1 = -3$?

Replace $y$ with $-2$ in the equation.

$$2y + 1 = -3$$

Replace $y$ with $-2$:

$$2(-2) + 1 = -3$$

$$-4 + 1 = -3$$

$$-3 = -3 \quad \text{True}$$

Since $-3 = -3$ is a true statement, $-2$ is a solution of the equation.
Determine whether a number is a solution:

Is 6 a solution of the equation $5x - 1 = 30$?

Replace $x$ with 6 in the equation.

$5x - 1 = 30$

$5(6) - 1 = 30$

$30 - 1 = 30$

$29 = 30$ False

Since $29 = 30$ is a false statement, 6 is not a solution of the equation.
To solve an equation, we will use properties of equality to write simpler equations, all equivalent to the original equation, until the final equation has the form

\[ x = \text{number} \quad \text{or} \quad \text{number} = x \]

Equivalent equations have the same solution.
The word “number” above represents the solution of the original equation.
Addition Property of Equality

Let \( a, b, \) and \( c \) represent numbers.

If \( a = b \), then

\[
\begin{align*}
    a + c &= b + c \\
    \text{and} & \\
    a - c &= b - c
\end{align*}
\]

In other words, the same number may be added to or subtracted from both sides of an equation without changing the solution of the equation.
Solve for \( x \).

\[ x - 4 = 3 \]

To solve the equation for \( x \), we need to rewrite the equation in the form \( x = \text{number} \).

To do so, we add 4 to both sides of the equation.

\[ x - 4 = 3 \]

\[ x - 4 + 4 = 3 + 4 \quad \text{Add 4 to both sides.} \]

\[ x = 7 \quad \text{Simplify.} \]
To check, replace $x$ with 7 in the original equation.

\[ x - 4 = 3 \quad \text{Original equation} \]
\[ 7 - 4 \neq 3 \quad \text{Replace } x \text{ with } 7. \]
\[ 3 = 3 \quad \text{True.} \]

Since $3 = 3$ is a true statement, 7 is the solution of the equation.
Helpful Hint

Remember to check the solution in the original equation to see that it makes the equation a true statement.
Remember that we can get the variable alone on either side of the equation. For example, the equations

\[ x = 3 \quad \text{and} \quad 3 = x \]

both have a solution of 3.
Multiplication Property of Equality

Let $a$, $b$, and $c$ represent numbers and let $c \neq 0$. If $a = b$, then

$$a \cdot c = b \cdot c \quad \text{and} \quad \frac{a}{c} = \frac{b}{c}$$

In other words, both sides of an equation may be multiplied or divided by the same nonzero number without changing the solution of the equation.
Solve for $x$

\[ 4x = 8 \]

To solve the equation for $x$, notice that 4 is \textit{multiplied} by $x$.

To get $x$ alone, we \textit{divide} both sides of the equation by 4 and then simplify.

\[
\frac{4x}{4} = \frac{8}{4} \quad \text{or} \quad 1 \cdot x = 2 \quad \text{or} \quad x = 2
\]
Check

To check, replace \( x \) with 2 in the original equation.

\[
4x = 8 \quad \text{Original equation}
\]

\[
4 \cdot 2 = 8 \quad \text{Let } x = 2.
\]

\[
? = 8 \quad \text{True}.
\]

The solution is 2.