5/12/14

$\log_x 2 = -\frac{1}{3}$, solve:

$x^{-\frac{1}{3}} = 2$

$\frac{1}{x^{\frac{1}{3}}} = 2$

$\frac{1}{\sqrt[3]{x}} = \frac{2}{1}$

$1 \cdot 1 = 2\sqrt[3]{x}$

$\frac{1}{2} = \frac{\cancel{2}\sqrt[3]{x}}{\cancel{2}}$

$\left(\frac{1}{2}\right)^3 = \left(\sqrt[3]{x}\right)^3$

$\boxed{\frac{1}{8}} = x$

12.6 #14

write a single log:

$2\log_9 x - \frac{1}{2}\log_9 x + 4\log_9 x$

$\log_9 x^2 - \log_9 x^{\frac{1}{2}} + \log_9 x^4$

$$\frac{x^2}{x^{\frac{1}{2}}} = x^{\frac{2}{1\cdot 2}-\frac{1}{2}} = x^{\frac{4}{2}-\frac{1}{2}} = x^{\frac{3}{2}}$$

$$\log_9 \frac{x^2}{x^{\frac{1}{2}}} + \log_9 x^4$$

$$\log_9 x^{\frac{3}{2}} + \log_9 x^4$$

$$\log_9 x^{\frac{3}{2}} \cdot x^4$$

$$\log_9 x^{\frac{3}{2}+\frac{4\cdot 2}{1\cdot 2}}$$

$$\log_9 x^{\frac{3}{2}+\frac{8}{2}}$$

$$\log_9 x^{\frac{11}{2}}$$

#12 $\log_5 x - \log_5(x+5) + \log_5(x^2-5)$

single log: $\log_5 \frac{x}{x+5} + \log_5(x^2-5)$

$$\log_5 \frac{x}{x+5} \cdot \frac{(x^2-5)}{1}$$

$$\boxed{\log_5 \frac{x^3-5x}{x+5}}$$

Solve: $\log_{11}(x^2+10) = 1$

$11^1 = x^2+10$

$11 = x^2+10$

$-10 \qquad -10$

$1 = x^2$

$\pm\sqrt{1} = \sqrt{x^2}$

$\boxed{\pm 1} = x$

Check:

12.5

$$\log_3 \frac{1}{243} = x$$

1st way

$$3^x = \frac{1}{243}$$

$$3^x = \frac{1}{3^5}$$

$$3^x = 3^{-5}$$

$$x = \boxed{-5}$$

2nd way

$$x = \log_3 \frac{1}{243} = \log_3 3^{-5} =$$

$$-5 \cdot \boxed{\log_3 3} = -5 \cdot 1 = -5$$

12.6 Write as a sum or difference of logarithms:

$$\log_2 \frac{7}{9z}$$

$$\log_2 7 - \log_2 9z$$

$$\log_2 7 - [\log_2 9 + \log_2 z]$$

$$\boxed{\log_2 7 - \log_2 9 - \log_2 z}$$

Write a sum or difference of logarithms

$$\log_9\left(\frac{x^5}{y}\right) = \log_9 x^5 - \log_9 y =$$

$$\boxed{5\log_9 x - \log_9 y}$$

If [illegible] $= 0.2$ and $\log_b$ [illegible] $= 0.5$

evaluate $\log_b 54 = \log_b (6$ [illegible]$) =$

$\log_b 6$ [illegible] $\log_b 9$

$0.2 + 0.5$

$= \boxed{0.7}$

Given [illegible] $= 0.4$, evaluate

$\log_b \sqrt[5]{2} = \log_b 2^{\frac{1}{5}} = \frac{1}{5} \cdot \log_b 2 =$

$\frac{1}{5} \cdot \frac{(0.4)}{1} =$

$$\frac{0.4}{5} = \boxed{0.08}$$

Graph : $y = 4^x$ and $y = \log_4 x$

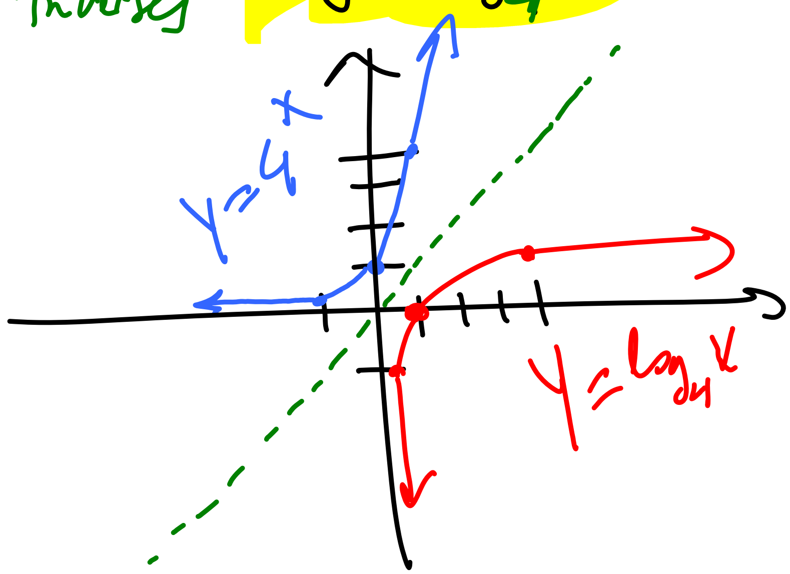
Inverses

x	y
-1	$\frac{1}{4}$
0	1
1	4

$y = 4^{-1} = \frac{1}{4}$

$y = \log_4 x$

x	y
1	0
$\frac{1}{4}$	-1



12.7 Common Logarithms, Natural Logarithms and Change of Base

Notation: $\log x$ means $\log_{10} x$

"Common Logarithm"

find

$$\log 10 = \log_{10} 10 = 1$$

$$\log \frac{1}{10} = \log_{10} 10^{-1} = -1 \cdot \log_{10} 10 = -1$$

$$\log \sqrt{10} = \log_{10} 10^{\frac{1}{2}} = \frac{1}{2}$$

$$\log 1 = 0$$

$$\log 1000 = \log_{10} 1000 = y$$

$$10^y = 1000$$

$$10^y = 10^3$$

$$y = 3$$

Another way: $\log_{10} 1000 = \log_{10} 10^3 = 3$

Approximate $\log 5 \approx 0.7$

number $e \approx 2.7$

$e^0 = 1$

$\ln x$ means $\log_e x$

"Natural Logarithm"

---

find $\ln e = \log_e e = 1$

$\ln e^2 = \log_e e^2 = 2$

$$\ln e^2 = 2 \cdot \ln e = 2 \cdot 1 = 2$$

find $\ln \frac{1}{e^3} = \ln e^{-3} = -3$

$-3 \cdot 1 = -3$

another way: $\ln \frac{1}{e^3} = \ln 1 - \ln e^3$

$0 - 3 \ln e$

$0 - 3 \cdot 1 = \boxed{-3}$

third way: $\ln \frac{1}{e^3} = y$

$$\log_e \frac{1}{e^3} = y$$

$$e^y = \frac{1}{e^3}$$

$$e^y = e^{-3}$$

$$y = \boxed{-3}$$

$$\ln \frac{1}{e^3} = \log_e \frac{1}{e^3} = \log_e e^{-3} = -3$$

---

appoximate $\ln 36 \approx 3.58$

---

evaluate $\ln\sqrt{e} + \log 100$

$\ln e^{\frac{1}{2}} \quad + \quad \log_{10} 10^2$

$\frac{1}{2}\ln e$

$\frac{1}{2}\cdot 1 \quad + \quad 2$

$\frac{1}{2} + \frac{2\cdot 2}{1\cdot 2} = \boxed{\frac{5}{2}}$

---

solve: $\ln 4x = 3$

$\log_e 4x = 3$

$$\frac{e^3}{4} = \frac{4x}{4}$$

exact [illegible] $\frac{e^3}{4} = x$

approximate to tenths: 5.0

---

solve: $\log x = 2.6$

$\log_{10} x = 2.6$

exact answer: $10^{2.6} = x$

approximate to four-decimal places

$x \approx 398.1072$

Solve: $\ln(2x-3) = 3.7$

exact answer, approximate to four-decimal places

e

$$e^{3.7} = 2x - 3$$

$+3 \qquad +3$

$$\frac{e^{3.7}+3}{2} = \frac{2x}{2}$$

Exact $\frac{e^{3.7}+3}{2} = x$

$$\boxed{x \approx 21.7237}$$

Change of base

$$\log_b x = \frac{\log x}{\log b} \quad \text{or} \quad \frac{\ln x}{\ln b}$$

ex evaluate $\log_2 7 = \frac{\log 7}{\log 2} \approx 2.8074$

ex $\log_{\frac{1}{3}} 8 = \frac{\log 8}{\log \frac{1}{3}} \approx -1.8928$

Continuous compounded interest formula

$$A = P \cdot e^{rt}$$

p = principal
r = % changed to decimal
t = number of years
A = total amount at the end

How much money does Barbara owe at the end of 4 years if 5% interest is compounded continuously on her $1600 debt?

$P = 1600$
$r = 0.05$
$t = 4$
$A = ?$

$A = P \cdot e^{rt}$

$A = 1600 \cdot e^{(0.05)(4)} =$

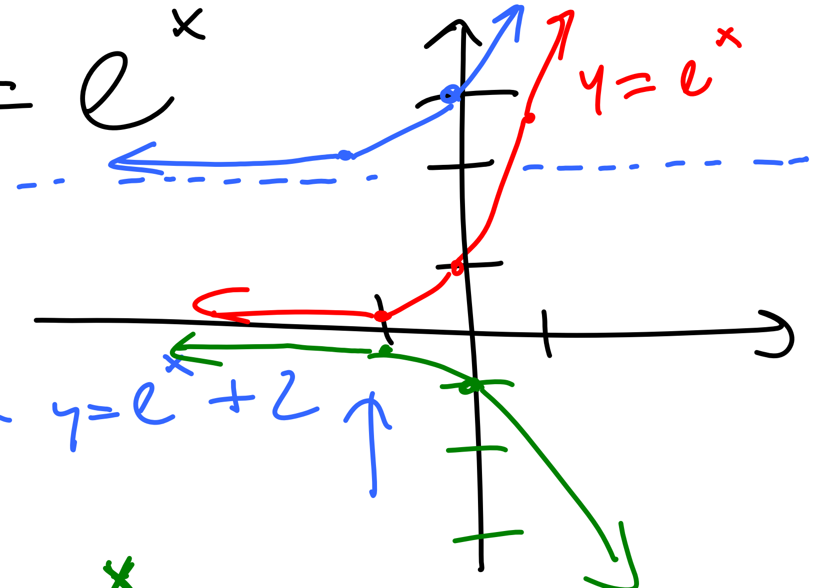
1600 exact answer

Round to nearest cents

$A \approx \$1954.29$

Graph $y = e^x$

x	y
-1	$\frac{1}{e} \approx 0.37$
0	1
1	$e \approx 2.7$



(b) Graph $y = e^x + 2$ ↑

(c) Graph $y = -e^x$: reflect to x-axis

Graph: $y = \ln x = \log_e x$

x	y
$0.4 \approx \frac{1}{e}$	-1
1	0
$2.7 \approx e$	1

$\log_e \frac{1}{e} = -1$