(5/7/14) $f(x) = \frac{2}{x-5}$, $f^{-1}(x) = ?$

① $y = \frac{2}{x-5}$

② switch x & y : $(y-5)x = \frac{2}{y-5} \cdot (y-5)$

③ Solve for y

$xy - 5x = 2$
$+5x \qquad +5x$

$\frac{xy}{x} = \frac{2+5x}{x}$

④ $f^{-1}(x) =$

$y = \frac{2+5x}{x}$

$$f^{-1}(x) = \frac{2+5x}{x}$$

Graph using transformations:

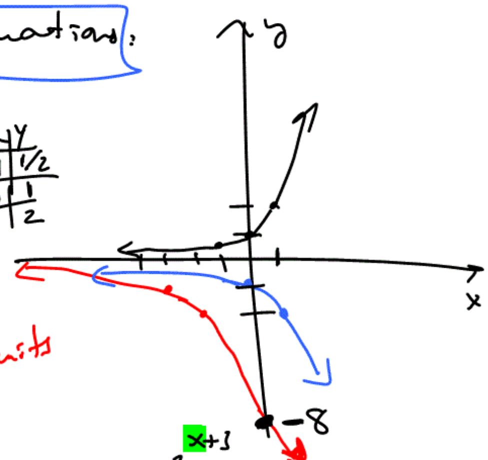
$f(x) = -2^{x+3}$

① $y = 2^x$

x	y
-1	1/2
0	1
1	2

② $y = -2^x$ reflect to x-axis

③ $y = -2^{x+3}$ left 3 units



⑥ find y-intercept of $y = -2^{x+3}$

$x = 0$

$y = -2^{0+3}$

$y = -2^3 = -8$

Solve: $\frac{1}{4} = 16^{2x-1}$

if we can, we need same bases

$4^{-1} = (4^2)^{2x-1}$

$4^{-1} = 4^{2(2x-1)}$

$4^{-1} = 4^{4x-2}$

$-1 = 4x - 2$
$+2 \quad\quad +2$

$1 = 4x$

$\boxed{\frac{1}{4}} = x$

$\frac{1}{a} = a^{-1}$

$a^{-1} = \frac{1}{a}$

$a^{-n} = \frac{1}{a^n}$

$(a^m)^n = a^{mn}$

That's what you missed, if you were absent !!!

if $a^x = a^y$

then $x = y$

(12.5) Logarithmic Functions

$\log_b X$ → inside of log

b: base

log = logarithm

$\log_b X$ does not mean $\log_b \cdot X$, not multiplication

$\log_b X$ "logarithm of X with base b"

"base b logarithm of X"

"log of X with base b"

$\log_2 8$

$\log_b X = y$ means $b^y = X$

---

example evaluate

$\log_2 8 = y$, need the y

$2^y = 8$

$2^y = 2^3$

$y = 3$

$\log_2 8 = 3$ ← because $2^{3} = 8$

---

find $\log_3 81 = y$

$$3^y = 81$$

$$3^y = 3^4$$

$$y = 4$$

$\log_3 81 = 4$ because $3^{4} = 81$

$\log_{71} 1 = ?$ $\quad 71^y = 1$

$y = 0$

$\log_{71} 1 = 0$ $\quad 71^0 = 1$

$\boxed{\log_b 1 = 0}$ because $b^0 = 1$

find $\log_{91} 91 = 1$ because $91^1 = 91$

$91^y = 91^1$

$\log_b b = 1$ $\quad b^1 = b$

$\log_b x$: $x > 0$, inside > 0
$b > 0$, base > 0

$\log_2(-5)$ undefined

$\log_2 \frac{1}{2} = y$ $\quad 2^y = \frac{1}{2}$

$2^y = 2^{-1}$

$= -1$ $\quad y = -1$

$\log_3 \sqrt{3} = y$ $\quad 3^y = \sqrt{3}$

$3^y = 3^{\frac{1}{2}}$

$= \frac{1}{2}$ $\quad y = \frac{1}{2}$

$\sqrt{a} = a^{\frac{1}{2}}$

$\sqrt[3]{a} = a^{\frac{1}{3}}$

$\sqrt[3]{a^2} = a^{\frac{2}{3}}$

$\log_4 \frac{1}{\sqrt[3]{4}} = ?$ $\quad 4^y = \frac{1}{\sqrt[3]{4}}$

$4^y = \frac{1}{4^{\frac{1}{3}}}$

$-\frac{1}{3}$

$4^y = 4^{-\frac{1}{3}}$

$y = -\frac{1}{3}$

$7^3 = 343$ write as logarithmic equation

$\log_b x = y \Rightarrow b^y = x$

$\log_7 343 = 3$

solve: $\log_x 27 = 3$

$\sqrt[3]{x^3} = \sqrt[3]{27}$

$x = 3$

Check it can't be negative or 0

---

solve: $\log_{16} x = \frac{1}{2}$

$16^{\frac{1}{2}} = x$

$\sqrt{16} = x$

$x = 4$

Check:

Solve: $\log_c \frac{1}{8} = \frac{1}{2}$

$c^{\frac{1}{2}} = \frac{1}{8}$

$(\sqrt{c})^2 = \left(\frac{1}{8}\right)^2$

$c = \boxed{\frac{1}{64}}$

Graph $y = \log_2 x$

x	y
$\frac{1}{4}$	-2
$\frac{1}{2}$	-1
1	0
2	1
4	2

$y = \log_2 \frac{1}{4} = -2$

$2^y = \frac{1}{4}$

$2^y = \frac{1}{2^2}$

$2^y = 2^{-2}$

$y = -2$

$y = \log_2 \frac{1}{2} = -1$

$2^y = \frac{1}{2}$

$2^y = 2^{-1}$

$y = -1$

$y = \log_2 1 = 0$

$2^y = 1$

$y = 0$

$y = \log_2 2 = 1$

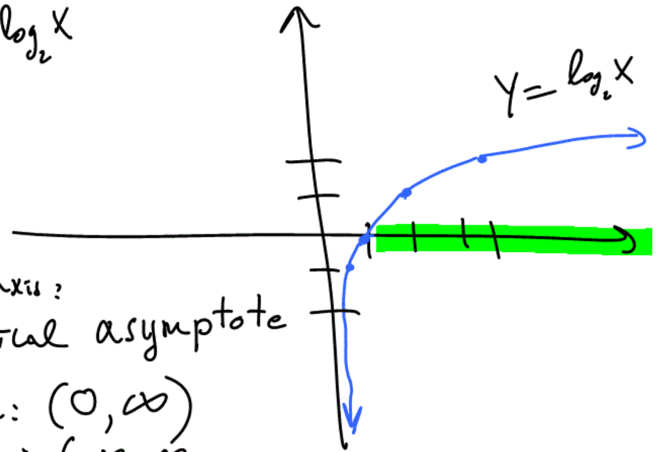
$2^1 = 2$

$y = \log_2 4 = 2$

$2^2 = 4$

Graph $Y = \log_2 X$

X	Y
$\frac{1}{4}$	-2
$\frac{1}{2}$	-1
1	0
2	1
4	2



Y-axis: vertical asymptote

domain: $(0, \infty)$

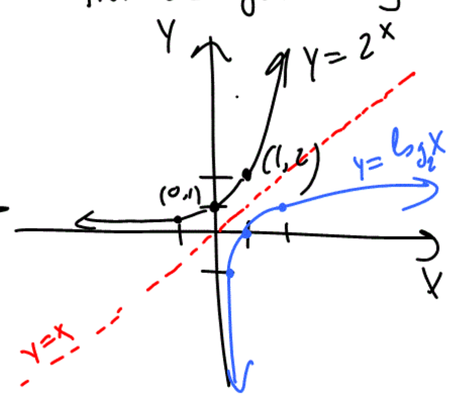
range: $(-\infty, \infty$

Graph $Y = \log_2 X$ } $Y = 2^X$

X	Y
$\frac{1}{4}$	-2
$\frac{1}{2}$	-1
1	0
2	1
4	2

X	Y
-2	$1/4$
-1	$1/2$
0	1
1	2

Inverse function



$$Y = \log_3 X, \quad f^{-1}(x) = ?$$

$$y = 3^x$$

$y = \left(\frac{1}{2}\right)^x$ base < 1

x	y
-1	2
0	1
1	$\frac{1}{2}$

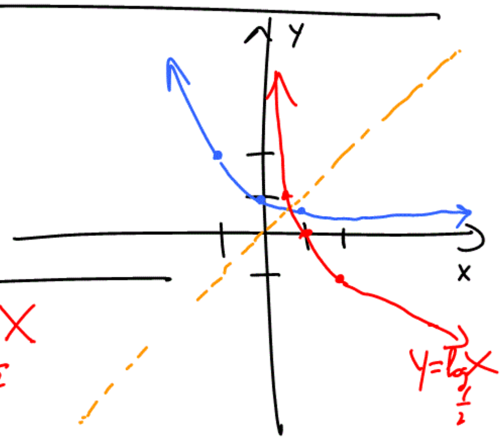
$y = \left(\frac{1}{2}\right)^{-1} = \frac{1^{-1}}{2^{-1}} = \frac{2^1}{1^1} = 2$

$y = \left(\frac{1}{2}\right)^0 = 1$

$y = \left(\frac{1}{2}\right)^1 = \frac{1}{2}$

$y = \log_{\frac{1}{2}} x$

x	y
2	-1
1	0
1/2	1



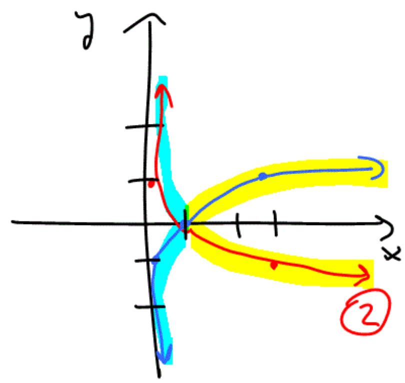
Graph using transformations

$y = -\log_3 x$

x	y
$\frac{1}{3}$	-1
1	0
3	1

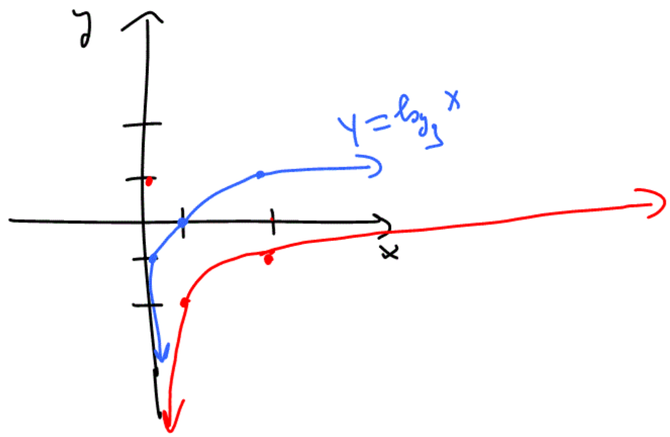
(1) $y = \log_3 x$

(2) $y = -\log_3 x$ reflect to x-axis



Use (1) & transformations to graph

$y = \log_3 x - 2$ ↓ $\quad y = \log_3 (x-2) \longrightarrow$



Properties of Logarithms

$\log_b 1 = 0$ | $\log_b b^x = x$

$\log_b b = 1$ | $b^{\log_b x} = x$ | $b^y = b^x$, $y = x$

$5^{\log_5 7} = 7$, $2^{\log_2 3} = 3$, $9^{\log_9 10} = 10$

Evaluate: $\log_3 3 + \log_5 \frac{1}{5} - \log_3 9 + \log_4 4^5$

$1 + (-1) - 2 + 5$

(3)

(12.6) Properties of Logarithms

① $\log_b X \cdot Y = \log_b X + \log_b Y$	$\log_2 3x = \log_2 3 + \log_2 x$ $\log_5 10 + \log_5 \sqrt{a} = \log_5 10\sqrt{a}$
② $\log_b \frac{X}{Y} = \log_b X - \log_b Y$	$\log_2 \frac{3}{2} = \log_2 3 - \log_2 2 = \log_2 3 - 1$ $\log_3 X^2 - \log_3 X = \log_3 \frac{X^2}{X} = \log_3 X$
③ $\log_b X^r = r \cdot \log_b X$	$\log_2 2^{\frac{1}{3}} = \frac{1}{3} \cdot \log_2 2 = \frac{1}{3} \cdot 1 = \frac{1}{3}$ $2 \log_3 \sqrt{3} = \log_3 (\sqrt{3})^2 = \log_3 3 = 1$

Write as a single logarithm and simplify if possible

(ex) $\log_6 2 + \log_6 3 = \log_6 2 \cdot 3 = \log_6 6 = 1$

(ex) $\log_9 6 + \log_9 (x^2+5) - \log_9 2$

$\log_9 6(x^2+5) - \log_9 2$

$$\log_9 \frac{\overset{3}{\cancel{6}}(x^2+5)}{\cancel{2}}$$

$$\log_9 3(x^2+5) \quad \text{OR}$$

$$\log_9 (3x^2+15)$$

Use the power rule:

$$\log_9{}^3 y = \log_9 y^{\frac{1}{3}} = \frac{1}{3} \cdot \log_9 y$$

Write as a single log:

$$6\log_6 x + 3\log_6 z$$

$$\log_6 x^6 + \log_6 z^3$$

$$\log_6 x^6 z^3$$

$6\log_6 x$ means

$6 \cdot \log_6 x$

Write as a single log:

$$2\log_6 x + \frac{1}{2}\log_6 x - 5\log_6 (x-1)$$

$$\log_6 x^2 + \log_6 x^{\frac{1}{2}} - \log_6 (x-1)^5$$

$$\log_6 \left(x^2 \cdot x^{\frac{1}{2}}\right) - \log_6 (x-1)^5$$

$$x^m \cdot x^n = x^{m+n}$$

$$\log_6 x^{\frac{5}{2}} - \log_6 (x-1)^5$$

$$\frac{2 \cdot 2}{1 \cdot 2} + \frac{1}{2} = \frac{4}{2} + \frac{1}{2} = \frac{5}{2}$$

$$\log_6 \frac{x^{\frac{5}{2}}}{(x-1)^5}$$