Chapter 10  Hypothesis Tests Regarding a Parameter

Ch. 10.1  The Language of Hypothesis Testing

Objective A: Set up a Hypothesis Testing

Hypothesis testing is a procedure, based on a sample evidence and probability, used to test statements regarding a characteristic of one or more populations.

- The null hypothesis $H_0$ is a statement to be tested.
- The alternate hypothesis $H_1$ is a statement that we are trying to find evidence to support.

Example 1:  Set up $H_0$ and $H_1$.

(a) In the past, student average income was $6000 per year. An administrator believes the average income has increased.

$H_0$: The average student income is $6000.
$H_1$: The average student income is greater than $6000.

or

$H_0$: $\mu = 6000$
$H_1$: $\mu > 6000$  Note: parameter being tested is a mean and it is a right tailed test.

(b) The percentage of passing a Math course was 50%. A Math professor believes there is a decrease in the passing rate.

$H_0$: Percentage of passing is 50%
$H_1$: Percentage of passing is less than 50%

or

$H_0$: $p= 0.5$
$H_1$: $p< 0.5$  Note: parameter being tested is a proportion and it is a left tailed test.

Objective B: Type I or Type II Error

Type I Error → Rejecting $H_0$ when $H_0$ is true.

Type II Error → Not rejecting $H_0$ when $H_1$ is true.

<table>
<thead>
<tr>
<th></th>
<th>REALITY</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>NULL HYPOTHESIS</td>
</tr>
<tr>
<td></td>
<td>TRUE</td>
</tr>
<tr>
<td>STUDY FINDINGS</td>
<td></td>
</tr>
</tbody>
</table>
| TRUE                 | ![Smiley face] | Type II error ($\beta$)
|                      |          | 'False negative' |
| FALSE                | ![Smiley face] | Type I error ($\alpha$)
|                      |          | 'False positive' |
Person is Innocent ($H_0$) | Person is Guilty ($H_1$)
--- | ---
Don’t reject null ($H_0$) (did not show evidence to show guilty) | Correct: no evidence to reject ($H_0$): Conclusion was declared innocent | Type II error: conclusion innocent but was guilty
Reject null ($H_0$) (did show evidence to show guilty) | Type I error: conclusion guilty but was innocent | Correct: evidence to reject ($H_0$): Conclusion was declared guilty

We use $\alpha$ for the probability of making Type I error.
We use $\beta$ for the probability of making Type II error.

For this statistics class, we will only control the Type I error. ($0.01 \leq \alpha \leq 0.10$)

$\alpha$ is also called the level of significance.

**Note:** If $\alpha$ is decreased, $\beta$ increases. Therefore, when your decrease type I error, type II error increases and vice versa. In other words, type I and type II errors are inversely related.

**Objective C: State Conclusions to Hypothesis Tests**

If $H_0$ is rejected, there **IS sufficient evidence** to support the statement in $H_1$.

If $H_0$ is NOT rejected, there **IS NOT sufficient evidence** to support the statement in $H_1$.

Example 1: In 2007, the mean SAT score on the reasoning test for all students was 710. A teacher believes that, due to the heavy use of multiple choice test questions, the mean SAT reasoning test has decreased.

(a) Determine $H_0$ and $H_1$. 
(b) Explain what it would mean to make a Type I error.

(Type I Error → Rejecting the $H_0$ when $H_0$ is actually true.)

Concluding that the mean is no longer 710 when in fact it still is.

(c) Explain what it would mean to make a Type II error.

(Type II Error → Not rejecting the $H_0$ when $H_1$ is true.)

Concluding that the mean is still 710 when it has actually decreased.

(d) State the conclusion if the null hypothesis is rejected.

There is enough evidence to support the teacher's claim that the mean SAT reasoning test score has decreased.

Example 2: The mean score on the SAT Math Reasoning exam is 516. A test preparation company states that the mean score of students who take its course is higher than 516.

(a) Determine the null and alternative hypotheses that would be used to test the effectiveness of the marketing campaign.

$H_0$: The mean SAT math score is 516.
$H_1$: The mean SAT math score is greater than 516.

or

$H_0$: $\mu = 516$
$H_1$: $\mu > 516$

(b) If sample data indicate that the null hypothesis should not be rejected, state the conclusion of the company.

There is not sufficient evidence to support the claim that the mean SAT math score is greater than 516.

(c) Suppose, in fact, that the mean score of students taking the preparatory course is 522. Has a Type I or Type II error been made?

Type II error: Not rejecting $H_0$ when we should have.
If we tested this hypothesis at $\alpha = 0.05$ level, what is the probability of committing a Type I error?

Since $\alpha$ is the probability of committing Type I error, there is a 5% chance of committing a Type I error.

(d) If we wanted to decrease the probability of making a Type II error ($\beta$), would we need to increase or decrease the level of significance?

If we increase the level of significance ($\alpha$), we decrease $\beta$ and vice versa. Therefore, we need to increase the level of significance $\alpha$ in order to decrease the probability of Type II error.

Example 3: According to the Centers for Disease Control, 15.2% of adults experience migraine headaches. Stress is a major contributor to the frequency and intensity of headaches. A massage therapist feels that she has a technique that can reduce the frequency and intensity of migraine headaches.

(a) Determine the null and alternative hypotheses that would be used to test the effectiveness of the massage therapist's techniques.

$H_0$: The percentage of adults who get migraines is 15.2%
$H_1$: The percentage of adults who get migraines is less than 15.2%.

or

$H_0$: $p = 0.152$
$H_1$: $p < 0.152$

(b) A sample of 500 American adults who participated in the massage therapists program results in data that indicate that the null hypothesis should be rejected. Provide a statement that supports the massage therapists program.

There is sufficient evidence to support the massage therapists’ claim that her technique reduces the percentage of migraine headache from 15.2%.

(c) Suppose, in fact, that the percentage of patients in the program who experience migraine headaches is 15.2%. Was a Type I or Type II error committed?

Type I error (Rejected $H_0$ when it was actually still true.)

Ch10.2 Hypothesis Tests for a Population Proportion

Objective A: Classical Approach
Z – Test for a population proportion

A hypothesis test involving a population proportion can be considered as a binomial experiment.

The best point estimate of \( p \), the population proportion, is a sample proportion, \( \hat{p} = \frac{x}{n} \).

There are two methods for testing hypothesis.

**Method 1: The Classical Approach Using \( z \) values (and comparing to 1, 2, or 3 SD’s)**

**Hypothesis Testing Using the Classical Approach**

If the sample proportion is too many standard deviations from the proportion stated in the null hypothesis, we reject the null hypothesis.

**Method 2: The \( p – \) Value Approach Using \( p \)-value (area)*

**Hypothesis Testing Using the \( p \)-Value Approach**

If the probability of getting a sample proportion as extreme or more extreme than the one obtained is small under the assumption the statement in the null hypothesis is true, reject the null hypothesis.

*We will briefly introduce the classical approach. The preferred method for this class is the \( p – \) value approach.

Testing Hypotheses Regarding a Population Proportion, \( p \).

Use the following steps to perform a Proportion \( Z \) – Test provided that

- The sample is obtained by simple random sampling or the data result from a randomized experiment.
- \( np_0(1 - p_0) \geq 10 \).
- The sampled values are independent of each other.

**Step 1** Determine the null and alternative hypotheses. The hypotheses can be structured in one of three ways:

<table>
<thead>
<tr>
<th>Two-Tailed</th>
<th>Left-Tailed</th>
<th>Right-Tailed</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H_0: p = p_0 )</td>
<td>( H_0: p = p_0 )</td>
<td>( H_0: p = p_0 )</td>
</tr>
<tr>
<td>( H_1: p \neq p_0 )</td>
<td>( H_1: p &lt; p_0 )</td>
<td>( H_1: p &gt; p_0 )</td>
</tr>
</tbody>
</table>

Note: \( p_0 \) is the assumed value of the population proportion.

**Step 2** Select a level of significance \( \alpha \), depending on the seriousness of making a Type I error.

**Classical Approach**

**Step 3** Compute the test statistic

\[
\hat{z}_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1 - p_0)}{n}}}
\]
Objective A: Classical Approach

Z – Test for a population proportion

Example 1: Use the classical approach to test the following hypotheses. We will not be using this method.

A study found that 25% of the population of drivers do not drive on the freeway. A researcher believes that this percentage is actually lower. He surveys 400 drivers and finds that 96 do not drive on the freeway. Test his claim at a significance level of \( \alpha = 0.05 \)

\[ n = 400; \quad x = 96; \quad \alpha = 0.05 \]

(a) Setup \( H_0 \) and \( H_1 \)

\( H_0: \) The percentage of non-freeway drivers is 25%.

\( H_1: \) The percentage of non-freeway drivers is less than 25%.

or

\( H_0: \) \( p = 0.25 \)

\( H_1: \) \( p < 0.25 \)

(b) Use the sample data of \( n = 400 \) and \( x = 96 \) to compute the test statistic and round your answer to four decimal places.

\[ \hat{p} = \frac{x}{n} = \frac{96}{400} = 0.24 \]

\[ z_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.24 - 0.25}{\sqrt{(0.25)(1-0.25)/400}} \]
\[ z_0 = \frac{0.24 - 0.25}{\sqrt{\frac{(0.25)(0.75)}{400}}} \approx -0.4619 \] *(z test statistic)*

(c) Use \( \alpha = 0.05 \) level of significance and StatCrunch to determine the critical value(s).
Use \( \alpha \) to find \( z_\alpha \) *(z critical value)*

\[ Stat \rightarrow Calculator \rightarrow Normal \rightarrow Input P(x \leq \_\_\_\_\_\_) = 0.05 \text{ (alpha)} \rightarrow Compute \]

Since the hypothesis test is ‘less than’, we are allowing for a total of 5% error on the left of the normal curve.

\[ Z_\alpha = -1.64. \text{ Note: We will not use this method. We can get these values through the hypothesis test. See examples below on hypothesis testing using StatCrunch.} \]

(d) Draw the \( Z \) – distribution that depicts the critical region and the \( Z \) – statistic.

\[ \alpha = 0.05 \]

\[ Z_\alpha = -1.64 \quad z_0 \approx -0.4619 \]

(e) What conclusion can be drawn? \text{The researchers’ results were not unusual. The null hypothesis } H_0 \text{ cannot be rejected.}

\text{Conclusion: There is not enough evidence to support the claim that the percentage of non-freeway drivers is less than 25%}.

**Ch10.3 Hypothesis Testing about a Population Mean with unknown \( \sigma \)**

**Objective A: Classical Approach**

\( t \) – test for a population mean

A \( t \)-test is performed (instead of a \( z \)-test when the standard deviation of the population is not known. For this section, we will not be given the population standard deviation so we will always perform a \( t \)-test.

It is rare in most cases to actually know the true population standard deviation.
Testing Hypotheses Regarding a Population Mean

To test hypotheses regarding the population mean, we use the following steps, provided that

- The sample is obtained using simple random sampling or from a randomized experiment.
- The sample has no outliers, and the population from which the sample is drawn is normally distributed or the sample size, \( n \), is large (\( n \geq 30 \)).
- The sampled values are independent of each other.

**Step 1** Determine the null and alternative hypotheses. The hypotheses can be structured in one of three ways:

<table>
<thead>
<tr>
<th>Two-Tailed</th>
<th>Left-Tailed</th>
<th>Right-Tailed</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H_0: \mu = \mu_0 )</td>
<td>( H_I: \mu &gt; \mu_0 )</td>
<td>( H_I: \mu &lt; \mu_0 )</td>
</tr>
<tr>
<td>( H_0: \mu \neq \mu_0 )</td>
<td>( H_I: \mu &lt; \mu_0 )</td>
<td>( H_I: \mu &gt; \mu_0 )</td>
</tr>
</tbody>
</table>

*Note: \( \mu_0 \) is the assumed value of the population mean.

**Step 2** Select a level of significance, \( \alpha \), depending on the seriousness of making a Type I error.

**Step 3** Compute the test statistic

\[
 t_0 = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}
\]

which follows Student’s \( t \)-distribution with \( n - 1 \) degrees of freedom.

**Step 5** State the conclusion.

The \( t \)-test procedure requires either that the sample was drawn from a normally distributed population or the sample size was greater than 30. Minor departures from normality will not adversely affect the results of the test. However, if the data include outliers, the \( t \)-test procedure should not be used. A normality plot is used to test whether the sample was drawn from a normally distributed population. A boxplot is used to detect outliers.

**Objective A**: Classical Approach (use test statistic \( t \) value ‘\( z \)’)

*\( t \)-test for a population mean*

We will not be using this method.

Example 1: Use StatCrunch to determine the critical value(s) and draw the distribution.

(a) a right-tailed test for a population mean at the \( \alpha = 0.05 \) level of significance based on a sample size of \( n = 18 \).

Stat \( \rightarrow \) Calculator \( \rightarrow \) \( t \) \( \rightarrow \) DF:17, \( P(x \geq \_\_\_\_\_\_\_\_\_\_) = 0.05 \) (alpha),
change the direction of the inequality to \( \geq \) for the right-tailed test \( \rightarrow \) Compute

\[ t_{\alpha} = 1.7396067 \]

Or about 1.74

Diagram:

(b) a left-tailed test for a population mean at the \( \alpha = 0.01 \) level of significance base on a sample size of \( n = 15 \).
Stat → Calculator → t → DF: 14, P (x ≥ ____) = 0.01 (alpha), check the direction of the inequality to ≤ for the left-tailed test → Compute

\[ t_{\alpha} = -2.6244941 \text{ or about } -2.62 \]

Diagram:

(c) a two-tailed test for a population mean at \( \alpha = 0.05 \) level of significance based on a sample size of \( n = 25 \).

Stat → Calculator → T → choose ‘between’ DF: 24, P (____<x <____) = 0.95 → Compute

\[ t_{\alpha} = \pm 2.0639 \]

Note that this is close to 2 SD’s. Diagram:

P-Value Approach (probability using area)

Ch10.2 Hypothesis Tests for a Population Proportion

Objective B: \( P – Value \) Approach

\( Z \) – Test for a population proportion

A hypothesis test involving a population proportion can be considered as a binomial experiment. (Two outcomes.)

The best point estimate of \( p \), the population proportion, is a sample proportion, \( \hat{p} = \frac{x}{n} \), provided the sample is obtained by

1) Simple random sampling
2) \( np_0(1 - p_0) \geq 10 \) to guarantee that a normal distribution can be used to test hypothesis for \( H_0 : p = p_0 \)
3) The sampled values are independent of each other \( (n \leq 0.05N) \). \( N \) is the size of the entire population.

Hypothesis Testing Using the P-Value Approach

If the probability of getting a sample proportion as extreme or more extreme than the one obtained is small under the assumption the statement in the null hypothesis is true, reject the null hypothesis.

Testing Hypotheses Regarding a Population Proportion, \( p \).

Use the following steps to perform a \( Z – Test \) for a Proportion,
**Example 1:** The percentage of physicians who are women is 27.9%. In a survey of physicians employed by a large university health system, 45 of 120 randomly selected physicians were women. Use the $P$-value approach to determine whether there is sufficient evidence at the 0.05 level of significance to conclude that the proportion of women physicians at the university health system exceeds 27.9%?

- $n = 120$, $x = 45$, $\alpha = 0.05$

(a) **Setup**

$H_0$: The percentage of female physicians is 27.9%.

$H_1$: The percentage of female physicians is greater than 27.9%.

or

$H_0: p = 0.279$

$H_0: p > 0.279$ (right-tailed test)

(b) Use StatCrunch to find $Z_0$ and its corresponding $P$-value.
Stat → Proportion Stats → one sample → with summary → # of successes 45, observations 120, value for \( p = 0.279 \), → select \( H_A: "p > \)” → compute

<table>
<thead>
<tr>
<th>Proportion</th>
<th>Count</th>
<th>Total</th>
<th>Sample Prop.</th>
<th>Std. Err.</th>
<th>Z-Stat</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>p</td>
<td>45</td>
<td>120</td>
<td>0.375</td>
<td>0.0409</td>
<td>2.344</td>
<td>0.0095</td>
</tr>
</tbody>
</table>

\[ Z_0 \approx 2.345 \quad \text{p-value} \approx 0.0095 \]

p-value 0.0095 < 5% so unusual

(c) Conclusion
If the ‘p’ is low, the null must go! We got unusual results since \( p = 0.94\% \) was less than 5%.
Therefore we can reject the null.
Conclusion:
There is enough evidence to support the claim that the percentage of female physicians is greater than 27.9%.
(d) Find the corresponding CI
Stat-prop. stat-one sample-with summary-#successes 45, observation 120-conf int. .95-compute CI: (0.289, 0.462) We are 95% that the true population proportion is between 28.9% and 46.2%
(e) How does this support the conclusion from the hypothesis test?
Since the claimed true population proportion of 27.9% is not in our CI, then there is evidence that the true population proportion in no longer 27.9%. In fact, our CI indicates that it is now greater.

Example 3: In 2000, 58% of females aged 15 years of age and older lived alone, according to the U.S. Census Bureau. A sociologist tests whether this percentage is different today by conducting a random sample of 500 females aged 15 years of age and older and finds that 285 are living alone.

Use the \( P \)-value approach and StatCrunch to determine whether there is sufficient evidence at the \( \alpha = 0.10 \) level of significance to conclude that the proportion has changed since 2000.

a. Set up
\[ H_0: \text{The percent of females living alone is 58\%.} \]
\[ H_1: \text{The percent of females living alone is not 58\%.} \]
or
\[ H_0: \ p = 0.58, \]
\[ H_1: \ p \neq 0.58 \]
b. Stat → Proportion Stats → One Sample → with summary → input # of successes 285, observations 500, select Hypothesis test value for p = 0.58, → H₁: p ≠ 0.58 → compute and record results

**Hypothesis test results:**

- p : Proportion of successes
- H₀ : p = 0.58
- H₁ : p ≠ 0.58

<table>
<thead>
<tr>
<th>Proportion</th>
<th>Count</th>
<th>Total</th>
<th>Sample Prop.</th>
<th>Std. Err.</th>
<th>Z-Stat</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>p</td>
<td>285</td>
<td>500</td>
<td>0.57</td>
<td>0.022072607</td>
<td>-0.45305024</td>
<td>0.6505</td>
</tr>
</tbody>
</table>

p-value ≈ 0.6505

Note: the sample proportion shown is found by 
\[ \hat{p} = \frac{285}{500} = 0.57 \]

c. Conclusion
This is not unusual since the probability of 0.6505 is greater than α = 0.10. Thus we cannot reject the null.

Conclusion:
There is not enough evidence to support the claim that the percent of females living alone is not 58%.

d. Find the corresponding confidence interval.
Stat-prop. stat-one sample-with summary-#successes 285, observation 500-conf int. .90-compute CI:(0.534, 0.606) We are 90% confident that the true population proportion is between 53.4% and 60.6%

(e) How does this support the conclusion from the hypothesis test?
Since the claimed true proportion of 58% is in this CI then there is no evidence that it has changed.

**P-Value Approach**

Ch10.3 Hypothesis Testing about a Population Mean with unknown \( \sigma \)

**Objective B: P – Value Approach**

\( t \) – test for a population mean

**DEFINITION** A *P-value* is the probability of observing a sample statistic as extreme or more extreme than the one observed under the assumption that the statement in the null hypothesis is true. Put another way, the *P*-value is the likelihood or probability that a sample will result in a statistic such as the one obtained if the null hypothesis is true.

*Statistical significance* means that the result observed in a sample is unusual when the null is assumed to be true.
The test procedure requires either that the sample was drawn from a normally distributed population or the sample size was greater than 30. Minor departures from normality will not adversely affect the results of the test. However, if the data include outliers, the \( t \)-test procedure should not be used. A normality plot is used to test whether the sample was drawn from a normally distributed population. A boxplot is used to detect outliers.
Example 1:
A survey of 15 large U.S. cities finds that the average commute time one way is 25.4 minutes. A chamber of commerce executive feels that the commute in his city is less and wants to publicize this. He randomly selects 25 commuters and finds the average is 22.1 minutes with a standard deviation of 5.3 minutes. At $\alpha = 0.01$, is there enough evidence to support the claim? Use the $P$-value approach.

(a) Setup
$$H_0: \text{The average commute time is 25.4 minutes.}$$
$$H_1: \text{The average commute time is less than 25.4 minutes.}$$

Or
$$H_0: \mu = 25.4$$
$$H_1: \mu < 25.4$$

Note: the population values are always stated in the hypothesis.

(b) Use StatCrunch to find $t_0$ and its corresponding $P$-value.

Notation: $\mu = 25.4$ (population mean) $n = 25$ (sample size)
$\bar{x} = 22.1$ (sample mean) $s = 5.3$ (sample standard deviation) $\alpha = 0.01$ (significance level)

Stat-t stats-one sample- with summary-sample mean (22.1), sample SD (5.3), sample size (25), hypothesis test = 25.4, '<' -compute

<table>
<thead>
<tr>
<th>Hypothesis test results:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$ : Mean of population</td>
</tr>
<tr>
<td>$H_0 : \mu = 25.4$</td>
</tr>
<tr>
<td>$H_A : \mu &lt; 25.4$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>MeanSample</th>
<th>MeanStd. Err.DF</th>
<th>T-Stat</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>22.1</td>
<td>1.06</td>
<td>24 -3.1132075 0.0024</td>
</tr>
</tbody>
</table>

$t_0 = -3.113$; $P$-value $\approx 0.0024$. This is an unusual result.

Since $P$-value $(0.0024) < 0.01$ ($\alpha$),

“If the $p$ is low, the null must go.”

Therefore, reject the $H_0$.

(c) Conclusion.: There is sufficient evidence to support the executive’s claim that the commute time in his city is less than 25.4 minutes.

(d) Compute the CI. How does this support your conclusion?

Since the significance level was 0.01, we will find the 99% confidence interval.

Stat-t stats-one sample- with summary-sample mean (22.1), sample SD (5.3), sample size (25), select conf. int., level 0.99 -compute

CI: (19.1, 25.1). The population mean time 25.4 is not in the CI. Therefore the mean time for the executive’s city is different. The CI supports that his city’s mean time is actually lower than 25.4 minutes. (This is the same conclusion as above using a hypothesis test.)

Example 4: Michael Sullivan, son of the author, decided to enroll in a reading course that allegedly increases reading speed and comprehension. Prior to enrolling in the class, Michael read
198 words per minute (wpm). The following data represent the words per minute read for 10 different passages read after the course.

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>206</td>
<td>217</td>
<td>197</td>
<td>199</td>
<td>210</td>
</tr>
<tr>
<td>210</td>
<td>197</td>
<td>212</td>
<td>227</td>
<td>209</td>
</tr>
</tbody>
</table>

(a) Because the sample size is small, we must verify that reading speed is normally distributed and the sample does not contain any outliers. Perform a normal probability plot (QQ plot) and a boxplot. Are the conditions for testing a \( t \) – hypothesis satisfied?

Enter data in column one

QQ plot: Graph → QQ Plot → var1 → Compute

![QQ plot image]

All points appear to be reasonably close to the line.

Boxplot:

Graph-boxplot-select var. 1-select ‘draw boxes horizontally-compute

![Boxplot image]

This supports that all points are reasonably close to the line since the boxplot shows no outliers. Yes, the conditions are met. Although the sample size is small, the distribution is roughly normally distributed.

(b) Was the class effective? Use the \( \alpha = 0.10 \) level of significance.
(i) Setup

\[ H_0: \text{His reading level is 198 wpm.} \]
\[ H_1: \text{His reading level is greater than 198 wpm.} \]

or

\[ H_0: \mu = 198 \]
\[ H_1: \mu > 198 \]

(ii) Use StatCrunch to find \( t_0 \) and its corresponding \( P \) value.

Stats → T Stats → one sample → with data → select var1 → select Hypothesis test for \( \mu \) → \( H_0: \mu = 198, H_A: \mu > 198 \) → compute

**Hypothesis test results:**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Sample Mean</th>
<th>Std. Err.</th>
<th>DF</th>
<th>T-Stat</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>var1</td>
<td>208.4</td>
<td>2.9672284</td>
<td>9</td>
<td>3.5049543</td>
<td>0.0033</td>
</tr>
</tbody>
</table>

\( t_0 = 3.505 \)
\( P\)-value \( \approx 0.0033 \)

P-value (0.0033) < \( \alpha \) (0.10). This is an unusual result. (If the \( P \) is low, the null must go.)

Reject \( H_0 \).

(iii) Conclusion.

There is sufficient evidence to support the claim that the reading course improves reading speed.

(iv) Compute the 90% CI. How does this support the above conclusion?

Stats → T Stats → one sample → with data → select var1 → select confidence interval for \( \mu \), .90 → compute

CI: (203.0 \( \pm \) 213.8), since 198 is not in the CI than his reading speed is no longer 198. The CI shows that it is higher. This supports the above conclusion.

**P-Value Approach**

**Ch10 Hypothesis Tests for a Population Standard Deviation** *(Supplemental Materials)*

**Objective B: \( P \)-Value Approach**

\( \chi^2 \)-Test about a population variance or standard deviation

The concepts are similar to the P-value approach for Ch10.2 and Ch10.3 except a Chi-Square distribution is used. To test hypotheses about the population variance or standard deviation, two conditions must be met:

1) the sample is obtained using simple random sampling and
2) the population is normally distributed. Recall: distribution is not symmetric and the values of are non-negative.

Example 1: A machine fills bottles with 64 fluid ounces of liquid. The quality-control manager determines that the fill levels are normally distributed with a mean of 64 ounces and a standard deviation of 0.42 ounce. He has an engineer recalibrate the machine in an attempt to lower the standard deviation. After the recalibration, the quality-control control manager randomly selects 19
bottles from the line and determines that the standard deviation is 0.38 ounce. Is there less variability in the filling machine? Use the level of significance $\alpha$ (0.05).

(a) Setup

$H_0$: The standard deviation is 0.42
$H_1$: The standard deviation is less than 0.42

Or

$H_0$: $\sigma = 0.42$
$H_1$: $\sigma < 0.42$

(b) Use StatCrunch to find the $P$-value.

Variance Stats $\rightarrow$ one sample $\rightarrow$ with summary $\rightarrow$ sample variance $0.38^2 = 0.1444$ $\rightarrow$ n = 19 $\rightarrow$

select Hypothesis test for $H_0$: $\sigma^2 = 0.42^2 = 0.1764$, $H_A$: $\sigma^2 < 0.1764$, $\rightarrow$ compute

<table>
<thead>
<tr>
<th>Variance</th>
<th>Sample Var.</th>
<th>Var. DF</th>
<th>Chi-Square Stat</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma^2$</td>
<td>0.1444</td>
<td>18</td>
<td>14.734694</td>
<td>0.3199</td>
</tr>
</tbody>
</table>

$P$-value (0.3199) is not less than $\alpha$ (0.05). It is not unusual. Cannot reject the $H_0$.

(c) What conclusion can be drawn?

There is insufficient evidence to support that the recalibration of the machine resulted in a lower standard deviation.

Example 2: Data obtained from the National Center for Health Statistics show that men between the ages of 20 to 29 have a mean height of 69.3 inches, with a standard deviation of 2.9 inches. A baseball analyst wonders whether the standard deviation of heights of major-league baseball players is less than 2.9 inches. The heights (in inches) of 20 randomly selected players are shown below.

{72 74 71 72 76 70 77 75 72 72 77 72 75 70 73 73 75 73 74 74}

$\mu = 69.3$ (population mean) $\sigma = 2.9$ (population standard deviation) $n = 20$ (sample size)

(a) Verify the data are normally distributed by drawing a normal probability plot.

Input raw data $\rightarrow$ Graph $\rightarrow$ QQ Plot $\rightarrow$ var1 $\rightarrow$ Compute

The sample data are close to the straight line. Thus, the data may have come from a population that is normally distributed.
(b) Use StatCrunch to compute the sample standard deviation.
Stat → Calculators → Summary Statistics → column → var1 → Compute : 2.06

(c) Test the notion at the $\alpha = 0.10$ level of significance.
$\mu = 69.3$ (pop) \hspace{1cm} \sigma = 2.9$ (pop) \hspace{1cm} $n = 20$ \hspace{1cm} $\bar{x} = 73.35$ (sample) \hspace{1cm} $\sigma_{\bar{x}} = 2.06$ (sample)

$\sigma^2 = 2.9^2 = 8.41$

Setup:
$H_0: \sigma = 2.9$
$H_1: \sigma < 2.9$
(use population SD)

P-Value:
Statcrunch: stat-variance stats-one sample-with data-select column var 1-hyp test $\sigma^2 = 8.41, \sigma^2 < 8.41$ – compute

From above $p$-value = 0.0374 which is less than the 0.10 significance level. This is an unusual result.
Therefore we reject the null $H_o$

Conclusion: There is sufficient evidence to support the claim that the standard deviation of heights of major-league baseball players is less than 2.9 inches

Homework 10.4 #5
For standard deviation enter data into statcrunch and use ‘summary statistics’
When asked to compute test statistic that is same as the chi square statistic for this section: get that from variance stats (with data for this problem) but make sure you use $\sigma^2$ for the population values
For p-value it is the same window: go to stat-variance stats-one sample-with data- then enter info

10.4 #2 For critical values that involve standard deviation use the chi-square calculators.
Careful for two tail test: if $\sigma = .05$ select between but enter .95 as your area
Note: statcrunch uses the variance so square sample and population standard deviations as necessary.