Math 265 Test 3 Review

1. Find the critical number(s), if any, of the function \( f(x) = e^{x^2 - x} \).

2. Find the absolute maximum and absolute minimum values, if any, of the function \( f(x) = x - \sqrt{x} \) on \([0, 25]\).

3. Find the number \( c \) that satisfies the conclusion of the Mean Value Theorem on the given interval.
   \[
   f(x) = 2\sqrt{x}, \quad [0, 25]
   \]

4. The function \( f(x) = \frac{1}{3} x^3 - 16x \) satisfies the hypotheses of Rolle’s Theorem on the interval \([0, 4\sqrt{3}]\). Find all values of \( c \) that satisfy the conclusion of the theorem.

5. Verify that the function satisfies the three hypotheses of Rolle's Theorem on the given interval. Then find all numbers \( c \) that satisfy the conclusion of Rolle's Theorem.
   \[
   f(x) = x^3 - 5x^2 + 6x + 2, \quad [0, 4]
   \]

6. Determine where the graph of the function \( f(x) = 9x - \sqrt{4 - x^2} \) is concave upward and where it is concave downward. Also, find all inflection points of the function.

7. How many points of inflection are on the graph of the function?
   \[
   f(x) = 18x^3 + 5x^2 - 12x - 20
   \]

8. Find the inflection points for the function given.
   \[
   f(x) = 5x + 2 - \sin x, \quad 0 < x < 3\pi
   \]

9. Find the critical numbers of the function.
   \[
   y = \frac{x}{x^2 + 36}
   \]

10. Find the limit.
    \[
    \lim_{t \to 0} \frac{4^t - 3^t}{t}
    \]
11. The graph of the derivative $f'(x)$ of a continuous function $f$ is shown. On what intervals is $f$ decreasing?

![Graph of $f'(x)$]

12. Find the limit.

$$\lim_{x \to \pi/2^-} \frac{-3}{5 \cos(x)}$$

13. Find the limit.

$$\lim_{x \to -\infty} \frac{x^2 - 2}{5x^2 + 9}$$

14. Sketch the curve.

$$y = \sqrt{\frac{x}{x - 1}}$$

15. Find the limit.

$$\lim_{x \to 0} \frac{x}{\tan^{-1}(2x)}$$
16. Sketch the curve. Find the equation of the slant asymptote.

\[ y = \frac{x^2}{x - 1} \]

17. The quantity demanded per month of an item is related to the unit price by the demand equation

\[ p = \frac{40}{0.05x^2 + 5}, \quad 0 \leq x \leq 20 \]

where \( p \) is measured in dollars and \( x \) is measured in units of a thousand. How many items must be sold by the manufacturer to maximize its revenue?

Hint: Recall that the revenue is given by \( R = px \).

18. The graph of the first derivative \( f'(x) \) of a function \( f \) is shown below. At what values of \( x \) does \( f \) have a local maximum or minimum?

![Graph of f'(x)](image_url)

19. Sketch the graph of the function \( f(x) = \frac{-3x^2}{x^2 + 1} \) using the curve-sketching guidelines.

20. Find the critical number(s), if any, of the function \( f(t) = 2t^4 - 4t^4 \).

21. Find the critical number(s), if any, of the function \( h(u) = \frac{7u}{u^2 + 36} \).
22. A manufacturer has been selling 1,200 television sets a week at $400 each. A market survey indicates that for each $30 rebate offered to the buyer, the number of sets sold will increase by 60 per week. Find the demand function.

23. The owner of a ranch has 4000 yd of fencing with which to enclose a rectangular piece of grazing land situated along a straight portion of a river. If fencing is not required along the river, what are the dimensions of the largest area he can enclose? What is the area?

24. A production editor decided that a promotional flyer should have a 1-in. margin at the top and the bottom, and a \( \frac{1}{2} \)-in. margin on each side. The editor further stipulated that the flyer should have an area of 392 in.\(^2\). Determine the dimensions of the flyer that will result in the maximum printed area on the flyer.

25. Sketch the graph of the function \( g(x) = \frac{x - 2}{x - 1} \) using the curve-sketching guidelines.

26. A piece of wire 10 m long is cut into two pieces. One piece is bent into a square and the other is bent into an equilateral triangle. How should the wire be cut for the square so that the total area enclosed is a minimum?

Round your answer to the nearest hundredth.

27. A rectangular storage container with an open top is to have a volume of 10 m\(^3\). The length of its base is twice the width. Material for the base costs $12 per square meter. Material for the sides costs $5 per square meter. Find the cost of materials for the cheapest such container.
28. Find two positive numbers whose product is 121 and whose sum is a minimum.

29. Find the slant asymptote of the function \( f(x) = \frac{x^2 + 7}{x} \).

30. Find the absolute maximum value of \( y = \sqrt{16 - x^2} \) on the interval \([-6, 6]\).

31. Estimate the value of \( \sqrt[3]{11} \) by using three iterations of Newton’s method to solve the equation \( x^3 - 11 = 0 \) with initial estimate \( x_0 = 2 \). Round your final estimate to four decimal places.

32. Use Newton's method with the specified initial approximation \( x_1 \) to find \( x_3 \), the third approximation to the root of the given equation. (Give your answer to four decimal places.)

\[ x^4 - 12 = 0, \quad x_1 = 6 \]

33. Use Newton's method to approximate the indicated root of \( x^4 + x - 4 = 0 \) in the interval \([1, 2]\), correct to six decimal places.

Use \( x_1 = 1.5 \) as the initial approximation.
Math 265 Test 3 Review
Answer Section

1. ANS: \( \frac{1}{2} \)

   PTS: 1  DIF: Medium  REF: 4.1.43  MSC: Bimodal
   NOT: Section 4.1

2. ANS:
   Abs. max. \( f(25) = 20 \)
   abs. min. \( f\left(\frac{1}{4}\right) = -\frac{1}{4} \)

   PTS: 1  DIF: Medium  REF: 4.1.47  MSC: Bimodal
   NOT: Section 4.1

3. ANS:
   \( c = \frac{25}{4} \)

   PTS: 1  DIF: Medium  REF: 4.2.13  MSC: Bimodal
   NOT: Section 4.2

4. ANS:
   4

   PTS: 1  DIF: Easy  REF: 4.2.2  MSC: Bimodal
   NOT: Section 4.2

5. ANS:
   \( c_1 = \frac{5}{3} + \frac{\sqrt{7}}{3}, \ c_2 = \frac{5}{3} - \frac{\sqrt{7}}{3} \)

   PTS: 1  DIF: Medium  REF: 4.2.2  MSC: Bimodal
   NOT: Section 4.2

6. ANS:
   CU on \((-2, 2),\)
   IP none

   PTS: 1  DIF: Difficult  REF: 4.3.47c  MSC: Bimodal
   NOT: Section 4.3
7. ANS: 1

PTS: 1  DIF: Medium  REF: 4.3.15  MSC: Bimodal
NOT: Section 4.3

8. ANS: $(\pi, 5\pi + 2), (2\pi, 10\pi + 2)$

PTS: 1  DIF: Medium  REF: 4.3.16  MSC: Bimodal
NOT: Section 4.3

9. ANS: 6, −6

PTS: 1  DIF: Medium  REF: 4.1.36  MSC: Numerical Response
NOT: Section 4.1

10. ANS: $\ln 4 - \ln 3$

PTS: 1  DIF: Medium  REF: 4.2.7  MSC: Bimodal
NOT: Section 4.4

11. ANS: $(4, 6) \cup (7, 9)$

PTS: 1  DIF: Medium  REF: 4.3.31a  MSC: Bimodal
NOT: Section 4.3

12. ANS: $-\infty$

PTS: 1  DIF: Medium  REF: 4.4.41  MSC: Bimodal
NOT: Section 4.4

13. ANS: $\frac{1}{5}$

PTS: 1  DIF: Easy  REF: 4.4.18  MSC: Bimodal
NOT: Section 4.4
14. ANS:

\[ y = x + 1 \]

PTS: 1  DIF: Medium  REF: 4.5.9  MSC: Bimodal

NOT: Section 4.5

15. ANS: \( \frac{1}{2} \)

PTS: 1  DIF: Medium  REF: 4.4.48  MSC: Numerical Response

NOT: Section 4.4

16. ANS:
17. ANS: 10,000
   PTS: 1 DIF: Difficult REF: 4.1.69 MSC: Short Answer
   NOT: Section 4.1

18. ANS: 3, 5, 8, 10
   PTS: 1 DIF: Medium REF: 4.3.31b MSC: Numerical Response
   NOT: Section 4.3

19. ANS:

   PTS: 1 DIF: Medium REF: 4.5.12 MSC: Short Answer
   NOT: Section 4.5

20. ANS: 0, \frac{4}{9}
   PTS: 1 DIF: Medium REF: 4.1.37 MSC: Bimodal
   NOT: Section 4.1

21. ANS: ±6
   PTS: 1 DIF: Medium REF: 4.1.35 MSC: Bimodal
   NOT: Section 4.1

22. ANS: \( p(x) = -0.5x + 1,000 \)
   PTS: 1 DIF: Medium REF: 4.7.61 MSC: Bimodal
   NOT: Section 4.7
23. ANS:
Dimensions: 2000 yd × 1000 yd, Maximum area: 2,000,000 yd²

PTS: 1  DIF: Medium  REF: 4.7.7  MSC: Short Answer
NOT: Section 4.7

24. ANS:
14 in. × 28 in.

PTS: 1  DIF: Difficult  REF: 4.7.32  MSC: Bimodal
NOT: Section 4.7

25. ANS:

PTS: 1  DIF: Medium  REF: 4.5.18  MSC: Bimodal
NOT: Section 4.5

26. ANS:
4.35 m

PTS: 1  DIF: Medium  REF: 4.7.35  MSC: Bimodal
NOT: Section 4.7

27. ANS:
$153.9

PTS: 1  DIF: Medium  REF: 4.7.16  MSC: Bimodal
NOT: Section 4.7

28. ANS:
11, 11

PTS: 1  DIF: Medium  REF: 4.7.3  MSC: Bimodal
NOT: Section 4.7
29. ANS: 
  \[y = x\]
  
  PTS: 1  DIF: Medium  REF: 4.5.61  MSC: Bimodal  
  NOT: Section 4.5

30. ANS: 
  4
  
  PTS: 1  DIF: Medium  REF: 4.1.52  MSC: Bimodal  
  NOT: Section 4.1

31. ANS: 
  2.224
  
  PTS: 1  DIF: Medium  REF: 4.8.12  MSC: Bimodal  
  NOT: Section 4.8

32. ANS: 
  3.4180
  
  PTS: 1  DIF: Medium  REF: 4.8.10  MSC: Numerical Response  
  NOT: Section 4.8

33. ANS: 
  1.283782
  
  PTS: 1  DIF: Medium  REF: 4.8.13  MSC: Numerical Response  
  NOT: Section 4.8