Module 4: Types of Statistical Studies

Create at least three examples of possible research questions (not given in OLI). Verify that your examples satisfy the three criteria for a well-stated research question.

- **Make an estimate about the population** (often an estimate about an average value or a proportion with a given characteristic)
  1. 
  2. 
  3. 

- **Test a claim about the population** (often a claim about an average value or a proportion with a given characteristic)
  1. 
  2. 
  3. 

- **Compare two populations** (often a comparison of population averages or proportions with a given characteristic)
  1. 
  2. 
  3. 

- **Investigate a relationship** between two variables in the population
  1. 
  2. 
  3. 

- **Focus on a cause-and-effect relationship**
  1. 
  2. 
  3. 

After your group has written fifteen questions, choose your most interesting question that best represents a well-stated research question about a population and write it below:

Use complete sentences to identify the following:

1. State the population.
2. State the variable (information you should collect from each individual in the sample).
3. State the numerical characteristic about the population related to the variable.
Module 5 collecting data

Martin works for a small store in the mall that makes pretzels. He recently came up with an idea to create a garlic mustard pretzel. The store manager is hesitant to offer this type of pretzel because they are not sure if their customers will buy it or not. Martin has had Statistics and offers to take a sample of customers and see if he can find approximately what percent of their customers would buy a garlic mustard pretzel. His boss tells him to go ahead, but that they do not have the funds for an expensive statistics study. Martin and his sales team brainstorm to come up with some possible ways to conduct the sample. For each sampling method, write a description of the method and an example of how Martin could use that method. Include how costly you think the method will be and whether or not it will represent the population of customers.

a. Voluntary Response Sampling

b. Random Sampling

c. Convenience Sampling

d. Census

The vice president at a community college wants to offer classes on Saturdays but is not sure if students will enroll in the course. He wants to conduct a study.

a) Write a research question.

b) Write a description of how each type of survey could be conducted for each of the following methods:

i. Voluntary Response sampling

ii. Random Sampling

iii. Convenience Sampling

iv. Census

Which method would you choose and why?

A car manufacturer wants to offer their new model in orange and purple striped color. They are not sure if the consumers are ready for this. They decide to conduct study.

c) Write a research question.

i. Write a description of how each type of survey could be conducted for each of the following methods:

ii. Voluntary Response sampling

iii. Random Sampling

iv. Convenience Sampling

v. Census

Which method would you choose and why?
Module 5 Sampling Techniques

Martin works for a small company that makes pretzels. He recently came up with an idea to create a garlic mustard pretzel. The company is hesitant to offer this type of pretzel because they are not sure if their customers will buy it or not. Martin has had Statistics and offers to take a sample of customers and see if he can find approximately what percent of their customers would buy a garlic mustard pretzel. His boss tells him to go ahead, but that they do not have the funds for an expensive statistics study. Martin and his sales team brainstorm to come up with some possible ways to conduct the sample. For each sampling method, write a description of the method and an example of how Martin could use that method. Include how costly you think the method will be and whether or not it will represent the population of customers.

1. Systematic Sampling

2. Voluntary Response Sampling

3. Cluster Sampling

4. Random Sampling

5. Convenience Sampling

6. Stratified Sampling
Module 6 Observational Studies verses Experiments

Directions: Analyze each of the following research questions. Tell whether the question is best answered through an observational study or from an experiment? Explain the reason for your choice. Now come up with a method for answering the research question. Make sure to include random sampling in your method and what population you will be addressing. If you chose to do an experiment, also describe some of the lurking variables in the situation and some ways that we can control these variables. Discuss the placebo and blinding technique and why they are important in this experiment?

1. Tuberculosis (TB) is a disease that affects millions of people worldwide. TB is a contagious bacterial infection that affects the lungs. Doctors have long speculated that Tuberculosis spreads the fastest in low income, crowded cities. Is there a relationship between low income, crowded cities and the number of cases of TB?

2. Dramamine is a common medication used in preventing and treating nausea, vomiting and dizziness caused by motion sickness. This medication has become a staple for thousands of people who travel by boat, car or plane. But is Dramamine really effective in preventing and treating the symptoms of motion sickness?

3. Unemployment has become a very important topic in the United States and worldwide. In an effort to create more jobs, many countries raise taxes on people’s income. Many argue that raising taxes will decrease people’s income and possibly force businesses to close down. It is your job to shed light on this issue. Is there a relationship between the tax rate percentage of a country and the unemployment rate?

4. College and High School students in the United States have long claimed that listening to music helps them study and retain information at a higher rate. But is this really true? Does listening to music really help a person better retain information?
Module 7 Distributions for Quantitative Data (Introduction/Dotplots/Histograms)

Please read the following pages from the OLI textbook:

Module 2 Introduction. What is Data, and Module 7 Dotplots and complete this worksheet. Please use complete sentences whenever possible.

Definitions (Write complete definitions of the following terms)

Data

Categorical variables

List two examples of a categorical variable.

Quantitative variable

List two examples of a quantitative variable.

What is the goal of data analysis?

Name two types of graphs discussed in this section that summarize the distribution of a quantitative variable.

1.

2.

To describe patterns in data, we use descriptions of ___________________, ___________________, ___________________, and ___________________.
List four shapes of a distribution and draw a quick sketch of each shape

1.

2.

3.

4.

What is another term used to describe the spread of a distribution?

What are the two ways to describe the spread of a distribution?

1.

2.

If you have any questions about the reading assignment, please write them here:
Please read the following pages from the OLI textbook:

Module 7 Histograms and complete this worksheet. Please use complete sentences whenever possible.

Definitions (Write complete definitions of the following terms)

Bins

Count (also called _________________)

Relative Frequency (write the formula to calculate it)

In which situation is a histogram most useful for displaying the distribution of a quantitative variable?

What are the disadvantages of using a histogram to display the distribution of a quantitative variable?

How does the bin size affect the appearance of the histogram?
Referring to the Oscar for Best Actress example, describe the overall structure of the paragraph using formal vocabulary to summarize the distribution of ages.

Sentence 1: What type of statement is this?

Sentence 2: What does the author describe in this sentence?

Sentence 3: What does the author describe in this sentence?

Sentence 4: What does the author describe in this sentence?

Sentence 5: What does the author describe in this sentence?

Read the comparison of distributions of birth weights for mothers who smoked/didn't smoke during pregnancy.

Were there any details in the comparison that you didn't understand? Please write your questions here:
Module 8 Measures of Center (Mean and Median)

Goals: Define the mean and median
Find the mean and median for a given data set.

These questions are based on your reading of OLI Module 8 Mean and Median.

Please define the following terms:

If you have \( n \) data values \( x_1, x_2, x_3, \ldots, x_n \), the mean (or \_____________) is

The median is \__________________________________________________________

\__________________________________________________________

The median divides the data into \________________________________________

If \( n \) is odd,

If \( n \) is even,

Example 1: Find the mean and median for each data set. Please show your use of the appropriate formula.

4, 6, 12, 5, 8
Mean \______________
Median \______________

Example 2: Find the mean and median for each data set. Please show your use of the appropriate formula.

10, 3, 17, 1, 8, 6, 12, 15
Mean \______________
Median \______________

Additional Practice

Find the mean and median for each data set. Round your answers to the nearest tenth.
Please show your use of the appropriate formula.

1. 8, 7, 8, 7, 8
Mean \______________
Median \______________
Please answer the following questions:

What effect does an outlier have on the mean of a distribution? Discuss both cases (when the shape is skewed to the left and when it is skewed to the right)

Under which condition(s) is the mean a better measure of center?

Under which condition(s) is the median a better measure of center?
Module 8 Mean and Median

Bill Gates Walks into a Diner
This example is used to illustrate the difference between the two different types of average: the mean and the median.

<table>
<thead>
<tr>
<th>Diner</th>
<th>Salary</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anna</td>
<td>$85,000</td>
</tr>
<tr>
<td>Bob</td>
<td>$50,000</td>
</tr>
<tr>
<td>Cathy</td>
<td>$45,000</td>
</tr>
<tr>
<td>Dave</td>
<td>$40,000</td>
</tr>
<tr>
<td>Eric</td>
<td>$30,000</td>
</tr>
<tr>
<td>Fran</td>
<td>$30,000</td>
</tr>
<tr>
<td>Gail</td>
<td>$35,000</td>
</tr>
</tbody>
</table>

1. Find the mean and median salary before Bill Gates walked into the diner.

2. Are the two measurements pretty similar? Would they represent a “typical” diner’s?

Now, Bill Gates walks in with annual income of, say, $1 billion, ($1,000,000,000)
3. Find the mean and median salary after Bill Gates walked into the diner.

4. Are the two measurements pretty similar? Would they represent a “typical” diner’s salary?

5. Should we be using the mean or median in each case? Why?

Module 8 Mean and Median (2)

1. As a group, find a data set with at least 10 numbers that has a mean of 13 and also a median of 13.

2. Add two numbers to your data set in number 1, so that the mean and median remain 13.

3. As a group, find a data set with at least 8 numbers where the mean is higher than the median. Find a data set with at least 8 numbers where the mean is lower than the median.

4. Make a data set with at least 6 numbers. Find the mean and median of the data set. Now add one large number to the data set. How do the mean and median change? Which changes more, the mean or the median?
Module 8  Summation Notation

Goals:  
- Use summation notation to express sums
- Expand and simplify expressions using summation notation

Summation Notation

Suppose you have a sum in which you are adding $n$ terms:

$$x_1 + x_2 + x_3 + \ldots + x_n$$

We can write sums more concisely if we use summation notation:

$$\sum_{i=1}^{n} x_i$$

We read this as “the sum of $x$ sub $i$ from $i$ equals 1 to $i$ equals $n$ ”

- $i$ = the index (the variable used in the algebraic expression)
- 1 = the starting integer (lower limit of summation)
- $n$ = the ending integer (upper limit of summation)

Partial sum: a finite number of terms are added

Note: The starting and ending numbers can be different values. Also, we may use other letters for the index (the most commonly used letters are $i$, $j$, $k$).

Example:

$$\sum_{k=3}^{n} x_k =$$

$$\sum_{i=0}^{5} x_i =$$

Write each sum using summation notation. Use the index $i$ and let $i$ begin at 1 in each summation.

1. $x_1 + x_2 + x_3 + \ldots + x_{12}$

2. $x_1 + x_2 + x_3 + \ldots + x_{125}$

Write out the sum and determine its value.

3. $\sum_{i=1}^{5} (x_i - 8)$

4. $\sum_{k=0}^{6} k^2$
Module 9 Quartiles and IQR

Activity 1: Calculating Quartiles, Range, $Q_1$, $Q_2$, $Q_3$, and IQR

For each of the following data sets, calculate the range, $Q_1$, $Q_2$, $Q_3$, and IQR for the following sets

1. $5, 7, 8, 8, 9, 11, 14, 16, 17, 19, 21, 25, 26, 29, 31, 33$
2. $2.1, 3.8, 5.1, 6.9, 7.2, 10.4, 11.3, 14.7, 15.1, 16.0$
3. $31, 34, 41, 52, 68, 71, 79, 83, 88, 90, 103$
4. $150, 152, 154, 155, 157, 159, 163, 164, 165$

Activity 2: Exploring Quartiles and IQR

1. Make a dot plot of the following data. Divide the dot plot into four groups with an equal amount of dots per group. Find approximate values of the quartiles. Make sure that 25% of the numbers are less than $Q_1$, 25% of the numbers are between $Q_1$ and $Q_2$, 25% of the numbers are between $Q_2$ and $Q_3$, and 25% of the numbers are greater than $Q_3$. Approximate the IQR.

$6, 6, 7, 7, 7, 9, 9, 11, 13, 13, 13, 15, 15, 15, 15, 16, 16, 19, 20, 21, 22, 22, 22$

2. Which data set has more spread? Why? Find approximate values of the quartiles. Approximate the distances between $Q_1$ and $Q_2$, and between $Q_2$ and $Q_3$ for data set A and data set B and compare.

3. Find two data sets with at least 15 different numbers that have the same median and the same range, but different IQR’s. Calculate the medians, ranges, quartiles and IQR to confirm that it works.
Module 9 Exploring Variability about the Median

For each of the following sets of data:

a) Find the Mean (round to one decimal place if needed)

b) Find the Median

c) Find the Range

d) Find the 5 point summary (minimum, maximum, Q₁, Q₂, Q₃)

e) IQR

f) Outliers

g) Create a boxplot

1. 4, 12, 4, 6, 20, 8, 13, 26

2. 27, 20, 12, 14, 25, 25, 26, 27, 22, 4

3. 16, 20, 14, 36, 20, 14, 15, 17, 17, 18, 18

4. 8, 5, 2, 5, 6, 9, 9, 6, 10, 8, 4, 7, 6, 6, 12, 8
Module 10 Exploring Variability about the Mean

Activity 1: Discover ADM

We will now be looking at a measure of spread from the mean. One measure of spread is often called the Average Distance from the Mean (or ADM for short). Look at the following data sets.

SET A: 2, 3, 5, 5, 6, 8, 10, 10, 11, 13, 15
SET B: 4, 4, 5, 6, 6, 15, 23, 24, 24, 29

1. Find the mean for each set.
2. Estimate the average distance of the numbers from the mean.
3. Try to find the exact value of the average distance of the numbers from the mean? (This is an actual number and not an estimation.)
4. Compare your computed ADM with what you estimated. How close did you get?
5. Can you recommend a procedure for finding the ADM?

Activity 2 Average Distance from the Mean

1. Calculate the Mean and the ADM for each of the following three data sets. Then answer the following questions.

Set A: 13, 11, 4, 21, 15, 8, 19, 17
Set B: 11, 8, 12, 10, 9, 13, 14, 10
Set C: 19, 15, 21, 23, 20, 16, 17, 21

Which data set had the highest center?
Which data set had the lowest center?
Which data set has the most spread?
Which data set has the least spread?

2. For each data set (A, B and C) above, use the mean and ADM to give an average value for the data set and two values that typical numbers in the data set fall in between.

3. Find a data set with at least 7 different numbers that has a mean of 20 and an ADM of 5.
For each of the following sets of data:

A. Find the Mean (round to one decimal place if needed)

B. Find the ADM

C. Find the Standard Deviation

D. Find the Quartiles

E. Using the mean what two numbers are the typical values in between

F. Using the median what two numbers are the typical values in between

1. 7, 12, 4, 6, 20, 8, 13, 26

2. 27, 20, 12, 14, 25, 23, 26, 27, 22, 4

3. 16, 24, 14, 36, 20, 14, 15, 17, 17, 18, 18

4. 8, 5, 2, 5, 6, 9, 9, 6, 10, 8, 4, 7, 6, 6, 13, 8
Module 11  Scatterplots, Linear Relationships and Correlation

1. Match each description (A, B, and C), of a set of measurements, to a scatterplot. Briefly describe your reasoning. Then describe what a dot represents in each graph.

   - **a.** \( x = \) average outdoor temperature and \( y = \) heating costs for a residence for 10 winter days
     What does a dot represent?

   - **b.** \( x = \) height (inches) and \( y = \) shoe size for 10 adults
     What does a dot represent?

   - **c.** \( x = \) height (inches) and \( y = \) score on an intelligence test for 10 teenagers
     What does a dot represent?

2. Researchers gathered data about the amount of fat, sugar, and carbohydrates in 22 fast food hamburgers. They gathered the information from fast food companies’ websites. To keep their measurements consistent, all data was described in grams. Using the companies' websites, the researchers also identified the number of calories for each hamburger. The researchers wanted to know if the amount of calories in the hamburgers depended on how much of each ingredient (fat, sugar and carbohydrates) was in the burger. That is, the researchers wanted to know whether there was a relationship between the amount of fat, sugar, and carbohydrates and the amount of calories in a hamburger. A line has been added to each graph to help you see the patterns more clearly.

   - **a.** About how many calories would you predict for a burger that has 20 grams of fat?

   - **b.** About how many calories would you predict for a hamburger that has 40 grams of carbohydrates?

   - **c.** Which prediction is likely to be more accurate? Why do you think this?

   - **d.** Which ingredient has the weakest impact on calories? Why do you think this?
e. What does the idea of strength tell you about whether an ingredient is a good predictor of calories?

f. What is the direction of the fat/calories graph? What does the direction of the line tell you about the association between the amount of fat and the calories in fast food hamburgers?

3. Suppose you gathered the following information from students at a local high school:

- GPA (grade point average),
- Average weekly hours spent working at a job,
- Average weekly hours spent doing homework,
- Average hours of sleep a night,
- Hourly wage,
- Height,
- Weight,
- Length of the left foot,
- Age of the oldest child in the student’s immediate family,
- Number of children in the student’s immediate family,
- Gender,
- Race, and
- Age.

From this list of variables, choose:

i. Two variables that you think will show a positive linear association,

ii. Two variables you think will show a negative linear association, and

iii. Two variables you think will not show an association in a scatterplot.

You may use the same variable for more than one comparison.

Explain why you chose the two-variable pairs. What was your reasoning for each pair?
1 Descriptions “A” and “B”, below describe a set of measurements in a scatterplot. The explanatory variable (x) is represented by the horizontal axis and the response variable (y) is represented by the vertical axis. Match each description to a scatterplot, and briefly explain your reasoning.

A $x = \text{city miles per gallons}$ and $y = \text{highway miles per gallon}$ for 10 cars
What does a dot represent?

B $x = \text{sodium (milligrams/serving)}$ and $y = \text{Consumer Reports quality rating}$ for 10 salted peanut butters
What does a dot represent?

2 These scatterplots show body measurements for 34 physically active adults. Match each description (A, B, and C) to a scatterplot. Briefly explain your reasoning.

A $x = \text{forearm girth (centimeters)}$, $y = \text{bicep girth (cm)}$. The forearm is the part of the arm between the elbow and wrist. The bicep is the part of the arm between the shoulder and elbow.
What does a dot represent?

B $x = \text{calf girth (cm)}$, $y = \text{bicep girth (cm)}$. The calf is the part of the leg below the knee. (Girth is the measurement around a body part.)
What does a dot represent?

C $x = \text{age (years)}$, $y = \text{bicep girth (cm)}$
What does a dot represent?
3 Match each description, A to F, to a scatterplot. Briefly explain your reasoning.

Scatterplot 1

Scatterplot 2

Scatterplot 3

Scatterplot 4

Scatterplot 5

Scatterplot 6

A \(x = \) month number (January = 1) and \(y = \) rainfall (inches) in Napa, California. Napa has several months of drought each summer.
What does each dot represent?

B \(x = \) month number (January = 1) and \(y = \) average temperature in Boston, Massachusetts. Boston has cold winters and hot summers.
What does each dot represent?

C \(x = \) year (from 1970) in five-year increments and \(y = \) Medicare expenditures ($). The yearly increase in Medicare costs has been getting bigger over time.
What does each dot represent?

D \(x = \) average temperature (°C) each month in San Francisco, California and \(y = \) average temperature (°F) each month in San Francisco, California.
What does each dot represent?

E \(x = \) chest girth (cm) and \(y = \) shoulder girth (cm) for a sample of men.
What does each dot represent?

F \(x = \) engine displacement (in liters) and \(y = \) city miles per gallon for a sample of cars. Engine displacement is roughly a measurement of the size of the engine. Larger engines tend to use more gas.
What does each dot represent?
Mod 12  Interpreting Slope and Y-intercept

For each of the following, you are given two variables and a linear equation relating those variables.

a) Write a sentence interpreting the y-intercept for given equation (include the units).

b) Write a sentence interpreting the slope for the given equation (include the units).

1. \( y = \text{number of owls in a town} \quad x = \text{number of barns in the town} \)
   \[ \text{Owls} = 18 + 5(\text{Barns}) \]

2. \( y = \text{number of cars parked on a street} \quad x = \text{number of houses on the street} \)
   \[ \text{Cars} = 12 + 2(\text{houses}) \]

3. \( y = \text{Cost in dollars for toner} \quad x = \text{number of pages printed} \)
   \[ \text{Cost} = 20 + .03(\text{page}) \]

4. \( y = \text{number of pixels on a computer screen} \quad x = \text{the width of the screen} \)
   \[ \text{Number of pixels} = 1300(\text{width}) + 12000 \]

5. \( y = \text{Gallons of milk cows on a farm produce per day} \quad x = \text{Acres of land available on the farm} \)
   \[ \text{Gallons of milk} = 1200(\text{acres of grass}) + 1000 \]

6. \( y = \text{the height in inches of male students in class} \quad x = \text{the height in inches of their fathers} \)
   \[ \text{Son's height} = 60 + .02(\text{father's height}) \]

7. \( y = \text{A company's the monthly cost of electricity in thousands of dollars} \quad x = \text{kilowatts of electricity used} \)
   \[ \text{Electricity bill} = 0.003(\text{kilowatt}) + 4 \]
1. Suppose that the water level of a river is 34 feet and that it is receding at a rate of 0.5 foot per day. Write an equation for the water level, $L$, after $d$ days. In how many days will the water level be 26 feet?

2. For babysitting, Nicole charges a flat fee of $3, plus $5 per hour. Write an equation for the cost, $C$, after $h$ hours of babysitting. What do you think the slope and the y-intercept represent? How much money will she make if she baby-sits 5 hours?

3. In order to "curve" a set of test scores, a teacher uses the equation $y = 2.5x + 10$, where $y$ is the curved test score and $x$ is the number of problems answered correctly. Find the test score of a student who answers 32 problems correctly. Explain what the slope and the y-intercept mean in the equation.

4. A plumber charges $25 for a service call plus $50 per hour of service. Write an equation in slope-intercept form for the cost, $C$, after $h$ hours of service. What is a reasonable domain for this situation?

5. Rufus collected 100 pounds of aluminum cans to recycle. He plans to collect an additional 25 pounds each week. Write and graph the equation for the total pounds, $P$, of aluminum cans after $w$ weeks. What does the slope and y-intercept represent? How long will it take Rufus to collect 400 pounds of cans?

6. A canoe rental service charges a $20 transportation fee and $30 dollars an hour to rent a canoe. Write and graph an equation representing the cost, $y$, of renting a canoe for $x$ hours. What is a reasonable domain for this situation?

7. A caterer charges $120 to cater a party for 15 people and $200 for 25 people. Assume that the cost, $y$, is a linear function of the number of $x$ people. Write an equation in slope-intercept form for this function. What does the slope represent? How much would a party for 40 people cost?
8. An attorney charges a fixed fee on $250 for an initial meeting and $150 per hour for all hours worked after that. Write an equation in slope-intercept form. Find the charge for 26 hours of work.

9. A water tank already contains 55 gallons of water when Baxter begins to fill it. Water flows into the tank at a rate of 8 gallons per minute. Write a linear equation to model this situation. Find the volume of water in the tank 25 minutes after Baxter begins filling the tank.

10. A video rental store charges a $20 membership fee and $2.50 for each video rented. Write and graph a linear equation $(y=mx+b)$ to model this situation. If 15 videos are rented, what is the revenue? If a new member paid the store $67.50 in the last 3 months, how many videos were rented?

11. Casey has a small business making dessert baskets. She estimates that her fixed weekly costs for rent and electricity are $200. The ingredients for one dessert basket cost $2.50. If Casey made 40 baskets this past week, what were her total weekly costs? Her total costs for the week before were $562.50. How many dessert baskets did she make the week before?

12. Tim buys a snow thrower for $1200. For tax purposes, he declares a linear depreciation (loss of value) of $200 per year. Let $y$ be the declared value of the snow thrower after $x$ years. What is the slope of the line that models this depreciation?

13. What is the y-intercept of the line.

14. Write a linear equation in slope-intercept form to model the value of the snow thrower over time.

15. What is a reasonable domain for this function?

16. Find the value of the snow thrower after 4.5 years.
Module 12  Fitting a Line

The following ordered pair data describes food trash, paper trash, plastic trash and total trash in tons. Use the following information to find the equation of the regression line that best fits the data. Then graph the line on the scatterplot. How well does the line fit the data? Use the following formulas

\[ y = a + bx \]
\[ b = r \frac{s_x}{s_y} \]
\[ a = \bar{y} - b\bar{x} \]

1) Scatterplot of TOTAL vs PAPER

Variable Mean StDev
PAPER 9.428 4.168
TOTAL 27.44 12.46

r = 0.729

2) Scatterplot of METAL vs PLAS

Variable Mean StDev
METAL 2.218 1.091
PLAS 1.911 1.065

r = 0.586

3) Scatterplot of TOTAL vs FOOD

Variable Mean StDev
FOOD 4.816 3.297
TOTAL 27.44 12.46

r = 0.583
Module 13 Finding and Interpreting r-squared

Use the given graphs and r-values to complete the following. Find the value of r-squared and write a sentence interpreting r-squared percentage in the context of data. Be sure to include the appropriate units. Now find possible other variables that may also account for the variability in y. What does this imply about making causal statements between the x variable and the y variable?

1. The x variable is describing the number of tons of paper trash and the y variable is the number of tons of total trash. \((r = 0.729)\)

2. The x variable is describing the number of tons of plastic trash and the y variable is the number of tons of metal trash. \((r = 0.586)\)

3. The x variable is describing the number of tons of food trash and the y variable is the number of tons of total trash. \((r = 0.583)\)

4. The x variable is describing the horsepower of an automobile and the y variable is describing the miles per gallon. \((r = -0.869)\)
Module 13 Finding and Interpreting r-squared Activity 2

For each of the following, you are given two variables and a correlation coefficient is given. Write two sentences interpreting the value of $r^2$. Try to be specific about other contributing factors.

8. $y = \text{number of owls in a town} \quad x = \text{number of barns in the town} \quad r = .65$

9. $y = \text{number of cars parked on a street} \quad x = \text{number of houses on the street} \quad r = .72$

10. $y = \text{Cost in dollars for toner} \quad x = \text{number of pages printed} \quad r = .92$

11. $y = \text{number of pixels on a computer screen} \quad x = \text{the width of the screen} \quad r = .96$

12. $y = \text{Gallons of milk cows on a farm produce per day} \quad x = \text{Acres of land available on the farm} \quad r = .55$

13. $y = \text{the height in inches of male students in class} \quad x = \text{the height in inches of their fathers} \quad r = .82$

14. $y = \text{A company's the monthly cost of electricity in thousands of dollars} \quad x = \text{kilowatts of electricity used} \quad r = .99$
Module 13 Activity #2

Making Residual Plots and Calculating the Standard Error of the Regression

1. For the following ordered pairs, the least squares regression equation is \( \hat{y} = 2.93 + 1.24x \)
   Make a scatter plot of the ordered pairs and draw the regression line on the scatterplot.
   Calculate the residuals \((y - \hat{y})\) for each x value and make a residual plot. Now square the residuals and find the SSE (sum of squared errors). Then find the standard error of the regression using the formula \( s_e = \sqrt{\frac{SSE}{n-2}} \)
   where n is the number of ordered pairs (Round your values two the nearest hundredth).

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>Predicted value ( \hat{y} )</th>
<th>Error (Residual)</th>
<th>( (Error)^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>3.17</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>4.41</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>5.17</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>11</td>
<td>6.94</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>7</td>
<td>7.67</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>12</td>
<td>9.41</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>8</td>
<td>10.14</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>15</td>
<td>11.87</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. For the following ordered pairs, the least squares regression equation is \( \hat{y} = 13.90 + 2.01x \)
   Make a scatter plot of the ordered pairs and draw the regression line on the scatterplot.
   Calculate the residuals \((y - \hat{y})\) for each x value and make a residual plot. Now square the residuals and find the SSE (sum of squared errors). Then find the standard error of the regression using the formula \( s_e = \sqrt{\frac{SSE}{n-2}} \)
   where n is the number of ordered pairs (Round your values two the nearest hundredth).

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>Predicted value ( \hat{y} )</th>
<th>Error (Residual)</th>
<th>( (Error)^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11</td>
<td>14.11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>15.81</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>13.90</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>7</td>
<td>15.91</td>
<td></td>
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<tr>
<td>5</td>
<td>5</td>
<td>14.91</td>
<td></td>
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<tr>
<td>6</td>
<td>1</td>
<td>12.90</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>13.90</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>15</td>
<td>13.90</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Module 14  Exponential Regression (1)

1. The number of people living in a city after $t$ years is given by: $y = 12000 \times (0.94)^t$
How many people were living in the city initially?
Is the population growing or decreasing?
At what rate?

2. A group of scientists observed a population of birds in a remote area and counted the birds in 2002.
After returning every year and counting the birds, they came up with the following formula for the number of
birds: $y = 14500 \times (1.04)^t$.
Is the population growing or decreasing?
At what rate?
How many birds used to live in the area in 2002?
Predict the population in 2010.

3. The initial number of bacteria in a sample is 156,000 and it grows at a rate of 14% every day.
Write an exponential equation for the number of bacteria after $t$ days.
Use your equation to predict the number of bacteria after 4 days.

4. The population in a small city is growing at a rate of 4.5% every year. If there are 18,000 people living in
the city now.
write an exponential equation for the population after $t$ years
Use our equation to predict the population after 5 years.

5. The bone mass in the average person’s legs decreases by 0.24 % every year. A person’s bone mass is
currently 5 kg.
Write an exponential equation for the mass after $t$ years
Use our equation to predict the mass after 10 years.
Module 14  Exponential Regression (2)

1. The number of people living in a city after \( t \) years is given by: \( y = 8000 \times (1.045)^t \)
   How many people were living in the city initially?
   What is the rate of growth for the city’s population?

2. A group of scientists observed a population of birds in a remote area and counted the birds in 1998.
   After returning every year and counting the birds, they came up with the following formula for the number of birds: \( y = 12000 \times (0.84)^t \).
   Is the population growing or decreasing?
   By what rate?
   How many birds used to live in the area in 1998?
   Predict the population in 2010.

3. The initial number of bacteria in a sample is 120,000 and it grows at a rate of 12% every day.
   Write an exponential equation for the number of bacteria after \( t \) days.
   Use your equation to predict the number of bacteria after 4 days.

4. The population in a small city is growing at a rate of 3.2% every year. If there are 15,000 people living in the city now.
   write an exponential equation for the population after \( t \) years
   Use our equation to predict the population after 8 years.

5. The bone mass in the average person’s legs decreases by .1% every year. A person’s bone mass is currently 3.4 kg.
   Write an exponential equation for the mass after \( t \) years
   Use our equation to predict the mass after 12 years.
Write an exponential equation for a regression model with the following initial amount and rate of change:

<table>
<thead>
<tr>
<th>Initial amount</th>
<th>Rate of change</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>5000</td>
<td>20% increase</td>
<td></td>
</tr>
<tr>
<td>5000</td>
<td>20% decrease</td>
<td></td>
</tr>
<tr>
<td>5000</td>
<td>2% increase</td>
<td></td>
</tr>
<tr>
<td>5000</td>
<td>2% decrease</td>
<td></td>
</tr>
<tr>
<td>5000</td>
<td>.2% increase</td>
<td></td>
</tr>
<tr>
<td>5000</td>
<td>.2% decrease</td>
<td></td>
</tr>
<tr>
<td>5000</td>
<td>.02% decrease</td>
<td></td>
</tr>
</tbody>
</table>

2. Find the percent increase or decrease for each of the following:

   a) \( y = 2000(1.03)^x \)

   b) \( y = 2000(1.003)^x \)

   c) \( y = 2000(.82)^x \)

   d) \( y = 2000(.982)^x \)
Module 14 Making Predictions with Exponential Functions

The following exponential functions were found to be reasonably good models for data sets. Use the functions and the scope of the data (domain) to make predictions. Remember if the x value is out of the scope of the data, do not make a prediction.

1. \( y = 24.6(1.075)^x \) (x value = the number of months since a local trash dump opened up, y value = number of tons of trash dumped, Domain: \( 0 \) months \( \leq x \leq 24 \) months)
   a) Predict the number of tons of trash dumped in the 6 month after opening.
   b) Predict the number of tons of trash dumped 1 year (12 months) after opening.
   c) Predict the number of tons of trash dumped 3 years (36 months) after opening.

2. \( y = 489.13(1.05)^x \) (x value = percent of pollutants removed from a lake, y value = cost of cleanup in dollars, Domain: \( 50 \leq x \leq 98 \).)
   a) Predict the cost to clean up 60 percent of the pollutants in the lake?
   b) Predict the cost to clean up 10 percent of the pollutants in the lake?
   c) Predict the cost to clean up 90 percent of the pollutants in the lake?

3. \( y = 1500(1.035)^x \) (x value = number of years since 1995, y value = amount in savings account, Domain: \( 0 \) years \( \leq x \leq 17 \) years)
   a) Predict the amount in the account in 2003 (x = 8)?
   b) Predict the amount in the account in 2025 (x = 30)?
   c) Predict the amount in the account in 2012 (x = 17)?

4. \( y = 71.175(1.016)^x \) (x value = age of bear in months, y value = weight of bear in pounds, Domain 8 mo \( \leq x \leq 177 \) mo)
   a) Predict the weight of a bear that is 5 years old (60 months)?
   b) Predict the weight of a bear that is 10 years old (120 months)?
   c) Predict the weight of a bear that is 20 years old (240 months)?
Module 14B Quadratic Relations (1)

1. A researcher has collected data on the price of gasoline from 1990 to 2010 and has found that the price in dollars after $t$ years can be predicted using the equation: $y = -0.0128x^2 + .3582x + 1.90$

   a) According to this model what was the price of gas in 1990?
   b) Using this model predict the price of gas 1998?
   c) Based on the equation, what year had the most expensive gas?
   d) How much did gas cost in that year?

2. The number of students attending Los Medanos community college from 2000 to present can be predicted using the equation: $y = 31.25x^2 - 500x + 12000$

   a) What direction does the parabola open?
   b) What year had the lowest enrollment?
   c) What was the enrollment the year?
   d) What was the enrollment in 2002?

3. Based on annual data collected by the Center of Disease Control, the number of cases of people infected with flu during any given month between September and December can be modeled using a quadratic equation: $y = -3x^2 + 14.4x + 12.9$ where $x = 0$ is September and $y$ is thousands of people.

   a) If possible predict the number of flu cases is November.
   b) If possible predict the number of flu cases in January.
   c) What month has the highest number of flu cases? How many people are infected at that time?

4. Match the graph to the equation:

   a) $y = 4(1.2)^x$
   b) $y = 5 - 6x + 2x^2$
   c) $y = 3 + 4x - 6x^2$
   d) $y = 5 - 2x$
   e) $y = 4(.87)^x$
Module 14B Quadratic Relations (2)

1. A researcher has collected data on the price of gasoline from 1995 to 2012 and has found that the price in dollars after t years can be predicted using the equation: \( y = -0.0130x^2 + .3582x + 1.96 \)

   a) According to this model what was the price of gas in 2002?
   b) Using this model predict the price of gas 1995?
   c) Based on the equation, what year had the most expensive gas?
   d) How much did gas cost in that year?

2. The number of students attending Glendale community college from 1990 to present can be predicted using the equation: \( y = 31.25x^2 - 375x + 8000 \)

   a) What direction does the parabola open?
   b) What year had the lowest enrollment?
   c) What was the enrollment the year?
   d) What is the prediction for their current enrollment?

3. Based on annual data collected by the Center of Disease Control, the number of cases of people infected with flu during any given month between September and February can be modeled using a quadratic equation: \( y = -3x^2 + 18x + 14.9 \) where \( x = 0 \) is September and \( y \) is thousands of people.

   a) If possible predict the number of flu cases is October.
   b) If possible predict the number of flu cases in January.
   c) What month has the highest number of flu cases? How many people are infected at that time?

4. Match the graph to the equation:

   i) \( y = 4(.89)^x \)
   ii) \( y = -1.2x + 6 \)
   iii) \( y = 4(1.12)^x \)
   iv) \( y = 3 - 4x + 5x^2 \)
   v) \( y = 15 + 6x - 2x^2 \)
The Basic Counting Principle

The basic counting principle is the method used to calculate the number of possibilities of events occurring. The events may be independent or dependent. Two events are considered to be independent if the occurrence of one does not affect the possibility of the occurrence of the other. For example, if we were to flip two coins simultaneously the outcome of one coin’s flip would not affect the other coin’s flip. So here we say that the two events are independent. Two events are said to be dependent if the occurrence of one does affect the possibility of the occurrence of the other. For example, if one card is drawn out of a deck of cards and not replaced the outcome of drawing a second card is affected by the drawing of the first since there is now one less cards in the deck.

| Basic Counting Principle | Suppose an event can occur in $m$ different ways and another event can occur in $n$ different ways. There are $m \times n$ ways that both events can occur. |

Example 1) Suppose that the XYZ clothing company has a line of shirts that come in three colors: blue, red and green. They also make the shirts in short sleeves and long sleeves. How many different styles do they make?

The tree diagram below shows all the possible styles that could make:

Notice that there are 6 different possibilities. Now by applying the counting principle to the same problem: there are two possibilities for the sleeve length and 3 possibilities for the color. According to the counting principle there are $2 \times 3 = 6$ total possibilities, which are apparent in our diagram.
Example 2) How many different three digit numbers can be formed using the numbers 2, 3 and 5 if each number can be used more than once?

The choices of the numbers for each digit are independent (that is, the choices are not affected by each other). Therefore, the number of choices remains three for each digit.

<table>
<thead>
<tr>
<th>Digits:</th>
<th>First</th>
<th>Second</th>
<th>Third</th>
</tr>
</thead>
<tbody>
<tr>
<td>Choices:</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

So, by the counting principle, there are $3 \times 3 \times 3 = 27$ different three digit numbers that can be formed by 2, 3 and 5.

Example 3) How many different three digit numbers can be formed using the numbers 2, 3 and 5 if each number can be used only once?

Notice that in this example the events are dependent. That means, the choices for each digit are affected by the previous choices made. If we use a number as the first digit, we may not use it again as the second or third digit. In this case we can choose either 2, 3 or 5 for the first digit, but once we decide on a number, only two of those will still be available for the second digit, and once that choice is made there will be only one remaining number for the last digit.

<table>
<thead>
<tr>
<th>Digits:</th>
<th>First</th>
<th>Second</th>
<th>Third</th>
</tr>
</thead>
<tbody>
<tr>
<td>Choices:</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

There are $3 \times 2 \times 1 = 6$ different possible three digit numbers that can be made without repeating a number.
Example 4) How many different three letter combinations can be formed using the letters in the word “CARD”, if each letter can be used only once?

<table>
<thead>
<tr>
<th>Letter:</th>
<th>First</th>
<th>Second</th>
<th>Third</th>
</tr>
</thead>
<tbody>
<tr>
<td>Choices:</td>
<td>4</td>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>

There are $4 \times 3 \times 2 = 24$ possible combinations.

Example 5) How many of the combinations in example 4 start with a vowel?

Here we only have one choice for our first letter (“A”), three choices for our second letter (“C”, “R” and “D”) and once one of these letters is chosen, there will be only two choices for the last letter.

<table>
<thead>
<tr>
<th>Letter:</th>
<th>First</th>
<th>Second</th>
<th>Third</th>
</tr>
</thead>
<tbody>
<tr>
<td>Choices:</td>
<td>1</td>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>

There are $1 \times 3 \times 2 = 6$ of the combinations start with a vowel.

**Factorial Notation:**

The **factorial** notation is used to represent a chain of multiplications where each number in the chain is one less than the previous number. In factorial notation, the product $3 \times 2 \times 1$ can be written as $3!$ (three factorial). This notation is commonly used in probability and statistics textbooks for convenience. For example $5! = 5 \times 4 \times 3 \times 2 \times 1$, and $7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$.

In general, $n$ factorial or $n! = n \times (n - 1) \times (n - 2) \times \cdots \times 3 \times 2 \times 1$.

For purposes of consistency in our calculations, we define $0! = 1$. 
Example 6) Evaluate \( \frac{8!}{5!} \)

\[
\frac{8!}{5!} = \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{5 \times 4 \times 3 \times 2 \times 1} = 8 \times 7 \times 6 = 336
\]

Example 7) Evaluate \( \frac{12!}{(10!)(2!)} \)

\[
\frac{12!}{(10!)(2!)} = \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{(10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1)(2 \times 1)} = \frac{12 \times 11}{2} = 66
\]

**Probability  Chapter 1 Practice Problems – The Basic Counting Principle**

1. In how many ways can the letters in the word "MATH" be used to make:

   a) a two–letter word with no letters repeating?

   b) a two–letter word with possible repeating letters?

   c) a two–letter word starting with a consonant and ending in a vowel?

2. A pizza parlor offers three different size pizzas, two different types of crust, and five different toppings.

   a) How many different one topping pizzas are possible?

   b) How many different one topping small pizzas are possible?
3. A California license plate starts with a non–zero digit, followed by three letters and then three numbers. How many different license plate combinations are possible?

4. A certain car model comes in blue, silver, black and red with either automatic or manual transmissions. In addition, a consumer has the option for air conditioning or not and a cassette player or a CD player. How many different choices do the consumers have?

5. In Sally’s closet, she has 4 different pairs of shoes, 6 different shirts, and 5 different pairs of pants. How many different outfits can she make?

6. Evaluate the following:

   a) $4!$

   b) $\frac{15!}{13!}$

   c) $\frac{8!}{(6!)(2!)}$

   d) $\frac{(4!)(7!)}{(6!)(5!)}$

   e) $\frac{3!}{0!}$
A permutation of objects is an arrangement of these objects in a certain order. To “permute” a set of objects means to arrange them in that order. As we saw in example 4 of the previous chapter, when we calculated the number of possible three letter words that could be formed using the letters in "CARD", we counted every possible permutation of three letters.

In a permutation, the order of the objects is very important. The arrangement of objects in a line is called a linear permutation.

If we want to calculate the number of ways we can choose 4 people from a group of 6 people to stand in a line, we are calculating a linear permutation. Applying the method from Chapter 1, we can find the number of possibilities as 6 × 5 × 4 × 3. Notice that

\[ 6 \times 5 \times 4 \times 3 \text{ is part of } 6!. \]

We can write an equivalent expression in terms of 6!.

\[ 6 \times 5 \times 4 \times 3 = \frac{(6 \times 5 \times 4 \times 3) \times 2 \times 1}{2 \times 1} = \frac{6!}{2!} \]

Note that the denominator is the same as (6 – 4)!.

The number of ways we can arrange 4 items chosen from 6 items in a line is written as \( P(6,4) \). In general, \( P(n,r) \) is read "n objects chosen r at a time" and is defined as follows:

\[ P(n,r) = \frac{n!}{(n-r)!} \]

Example 1) The CSUN history club which has 25 members is holding elections for a president and a vice–president. In how many possible ways can the club fill these positions?

\( n = 25 \) and \( r = 2 \)

\[ P(25,2) = \frac{25!}{(25-2)!} = \frac{25!}{23!} = \frac{25 \times 24 \times 23!}{23!} = 25 \times 24 = 600 \]
Example 2) In how many ways can a store manager arrange 2 different blue dresses, 5 different green dresses, and 4 different red dresses on a straight rack if the same color dresses have to be displayed together?

We first need to calculate in how many ways the dresses of the same color can be arranged next to each other.

The blue dresses can be arranged in \( P(2,2) = \frac{2!}{(2-2)!} = \frac{2!}{0!} = \frac{2}{1} = 2 \) different ways.

The green dresses can be arranged in \( P(5,5) = 120 \) different ways.

The red dresses can be arranged in \( P(4,4) = 24 \) different ways.

In addition to these choices, the store manager can also group the colors in a different order (blue, green, red as opposed to red, blue, green). This is \( P(3,3) = 6 \) different ways.

So by the basic counting principle, she has \( 2 \times 120 \times 24 \times 6 = 34,560 \) different ways of displaying the dresses.

In example 1 of this chapter, we considered the number of possibilities for choosing a president and vice–president from a group of 25. We used a permutation because order mattered. Suppose Mary and John were elected. There are two situations: either Mary is President and John is Vice–President or John is President and Mary is Vice–President. Now suppose instead that the same club wants to choose two representatives for the homecoming committee. In this case, the people are being chosen without regard to any order. So if Mary and John are chosen, there is no need to consider two situations. Such a selection is called a combination. So in combinations the order does not matter while in permutations the order is important.

We use the notation \( C(n,r) \) when choosing \( r \) objects from a group of \( n \) objects when order does not matter. This is generally read "\( n \) choose \( r \)."

\[
C(n,r) \text{ or } _nC_r = \frac{n!}{r!(n-r)!}
\]

Example 3) In how many ways can two members be chosen to represent the CSUN history club (from example 1) on the homecoming committee?
\( n = 25 \) and \( r = 2 \)

\[
\binom{25}{2} = \frac{25!}{2!(25-2)!} = \frac{25 \times 24 \times 23!}{2 \times 23!} = \frac{25 \times 24}{2} = 300
\]

Compare this answer with the answer in example 1.

**Practice Problems – Probability Chapter 2**

1. In how many ways can 6 different books be arranged on a shelf?

2. From a committee of 9 people, in how many ways can we choose a chairperson, a secretary, and a treasurer?

3. The CSUN Art Gallery has 12 paintings submitted for display. In how many ways can they choose 4 of the paintings to be displayed?

4. In how many ways can a committee of 3 people be chosen from a group of 8 people?

5. A combination lock has 15 positions labeled. It can be unlocked by moving the dial first to the right, then to the left, and then back to the right. How many 3 number combinations are possible if no number is used more than once?

6. An ice cream parlor has 31 flavors of ice cream.
a) How many different triple scoops are possible if no flavor is used more than once?

b) How many different triple scoops on a cone are possible if no flavor is used more than once? (Note – a Vanilla/Chocolate/Strawberry cone is different than a Strawberry/Chocolate/Vanilla cone.)

7. Ten students are to be divided into two groups of 4 and 6. In how many ways can this be done?

8. A catering company offers five different appetizers, 3 different main courses, and 2 different desserts. How many different menus are possible if you are allowed 2 appetizers, 1 main course, and 1 dessert for the meal?

9. A video clerk needs to arrange the new arrival shelf. There are 4 new comedies, 3 new horror movies, and 2 new action films. In how many ways can he arrange the movies on a shelf if he needs to keep them grouped by type?
Probability Chapter 3

Probability is loosely defined as likelihood of the success or failure of obtaining a desired outcome. Knowledge of how to compute probabilities is necessary in many different professional areas, including physical, biological and social sciences, and journalism, among others, and is useful in daily life. Probability theory is often used to make predictions. For example, we can use probability theory to make predictions about the average blood pressure a human being will have at a certain age, the odds of rolling a seven with two fair dice, or the chances of winning the lottery. Such random events cannot be predicted with certainty, but the relative frequency with which they occur in a long series of trials is often remarkably stable. This fact enables us to compute probabilities about such events.

The probability of a favorable outcome is defined as the number of favorable outcomes divided by the number of possible outcomes, where a favorable outcome is the result we are seeking and a possible outcome is any possible result.

| Probability of event E | P(E) = \[
\frac{\text{number of ways E can occur}}{\text{number of all possible outcomes}}\] |

Example 1) What is the probability that a fair coin comes up heads?

In this problem, the favorable outcome is heads on a fair coin. The number of favorable outcomes is 1 (there is only 1 way to get heads on a fair coin). The number of possible outcomes (either heads or tails) is 2. So the probability of obtaining heads is $\frac{1}{2}$.

Example 2) Find the probability of rolling a two on a fair die.

Here, the favorable outcome is obtaining a two on a fair die. The number of favorable outcomes is 1 (there is only 1 way to get a two on a fair die). The number of possible outcomes is 6. So the probability of obtaining a two on a fair die is $\frac{1}{6}$. 
Example 3) Two fair dice are thrown, find the probability that a total of 7 is rolled.

Let's use the following table to help us determine the different possibilities. The horizontal rows represent the faces of one die and the vertical columns represent the faces of the other die. The body of the table, then, gives all possible outcomes of rolling the two dice.

<table>
<thead>
<tr>
<th>First die</th>
<th>Second die</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
</tr>
</tbody>
</table>

Notice that there are six different ways of rolling a 7 and there are 36 possible outcomes given in the body of the table. So the probability of rolling a seven is \( \frac{6}{36} = \frac{1}{6} \).

Example 4) In a family of 2 children, what is the probability that both children are girls?
We will denote B for boy and G for girl. There are 4 possible outcomes: BB, BG, GB, GG. Our favorable outcome is GG. There is only 1 way for this to happen. So the probability of two girls is $\frac{1}{4}$.

Now suppose we are interested in calculating the probability of two different events that occur simultaneously. If the two events are independent, then the probability of both events occurring is the product of the probability of each individual event’s occurrence.

<table>
<thead>
<tr>
<th>Probability of Two Independent Events</th>
<th>If two events, A and B, are independent, then the probability of both events occurring is found by the following: $P(A \text{ and } B) = P(A) \times P(B)$</th>
</tr>
</thead>
</table>

Example 5) A fair die is tossed three times. Find the probability that it comes up 6 every time.

Since the outcomes of each toss of the die are independent events, we can find the probably of three consecutive 6’s by multiplying the individual probabilities: $\frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{216}$.

Example 6) A bag contains 18 blue marbles and 6 red marbles. A marble is selected at random, and placed back in the bag, then a second marble is selected. Find the probability of the first marble being red and the second marble being blue.

In this example, the marble is replaced back in the bag, therefore the two events are independent. The probability of a red marble being drawn is $\frac{6}{24}$ and the probability of a blue marble being drawn is $\frac{18}{24}$. So the probability of a red marble and a blue marble is found by: $P(\text{Red and Blue}) = \frac{6}{24} \times \frac{18}{24} = \frac{108}{576} = \frac{3}{16}$.
Practice Problems – Probability Chapter 3

1. Find the probability of drawing a red card from a regular 52-card deck.

2. A wheel has 9 sections numbered 1 through 9. Suppose that you spin the wheel three times. Find the following probabilities:
   a) Spinning an even number on the first spin.
   b) Spinning a number divisible by 4 on the first spin.
   c) Spinning three odd numbers in a row.
   d) Spinning a multiple of 3 on the first spin and a multiples of 4 on the second and third spin.

3. Given a deck of cards with 52 cards, a card is drawn. Find the following probabilities:
   a) A face card (Jack, Queen or King) being drawn.
   b) An Ace being drawn.
   c) A 5 of hearts being drawn twice after it has been placed back in the deck and the deck is shuffled before the second drawing.
4. A coin is tossed 4 times. What is the probability of it coming up tails every time.

5. If two dice are thrown,
   a) What is the probability of rolling double 6's?
   b) What is the probability that the roll totals 9?
Module 15  Two Way Tables (1)

Creating a two way table

Directions: Here is some data taken from the medical records department at a local hospital. The data includes age, gender, blood type (A, B, AB, O), Rhesus factor (Rh + or Rh -) and part of the hospital the patient was in (Medical/Surgical, Intensive Care Unit, Same Day Surgery, Emergency Room).

1. Create a two way table that we could use to compare gender to bloodtype.

2. Create a two way table that we could use to compare the part of the hospital the patient was in to the patient's age (18-35, 36-49, 50-64, 65 or above).

3. Create a two way table that we could use to compare blood type to Rh factor.

<table>
<thead>
<tr>
<th>Patient ID#</th>
<th>Age</th>
<th>Gender</th>
<th>Blood Type</th>
<th>Rh Factor</th>
<th>Floor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>23</td>
<td>M</td>
<td>A</td>
<td>-</td>
<td>SDS</td>
</tr>
<tr>
<td>2</td>
<td>68</td>
<td>M</td>
<td>O</td>
<td>+</td>
<td>ER</td>
</tr>
<tr>
<td>3</td>
<td>51</td>
<td>F</td>
<td>AB</td>
<td>+</td>
<td>Med/Surg</td>
</tr>
<tr>
<td>4</td>
<td>74</td>
<td>M</td>
<td>O</td>
<td>-</td>
<td>ICU</td>
</tr>
<tr>
<td>5</td>
<td>49</td>
<td>F</td>
<td>O</td>
<td>+</td>
<td>SDS</td>
</tr>
<tr>
<td>6</td>
<td>62</td>
<td>F</td>
<td>O</td>
<td>+</td>
<td>Med/Surg</td>
</tr>
<tr>
<td>7</td>
<td>35</td>
<td>M</td>
<td>A</td>
<td>+</td>
<td>SDS</td>
</tr>
<tr>
<td>8</td>
<td>46</td>
<td>F</td>
<td>O</td>
<td>+</td>
<td>Med/Surg</td>
</tr>
<tr>
<td>9</td>
<td>72</td>
<td>F</td>
<td>O</td>
<td>+</td>
<td>ER</td>
</tr>
<tr>
<td>10</td>
<td>61</td>
<td>M</td>
<td>B</td>
<td>+</td>
<td>SDS</td>
</tr>
<tr>
<td>11</td>
<td>43</td>
<td>F</td>
<td>A</td>
<td>-</td>
<td>Med/Surg</td>
</tr>
<tr>
<td>12</td>
<td>81</td>
<td>M</td>
<td>O</td>
<td>+</td>
<td>ICU</td>
</tr>
<tr>
<td>13</td>
<td>65</td>
<td>M</td>
<td>A</td>
<td>+</td>
<td>Med/Surg</td>
</tr>
<tr>
<td>14</td>
<td>59</td>
<td>F</td>
<td>O</td>
<td>-</td>
<td>SDS</td>
</tr>
<tr>
<td>15</td>
<td>44</td>
<td>F</td>
<td>B</td>
<td>+</td>
<td>ICU</td>
</tr>
<tr>
<td>16</td>
<td>26</td>
<td>M</td>
<td>O</td>
<td>+</td>
<td>ER</td>
</tr>
<tr>
<td>17</td>
<td>58</td>
<td>F</td>
<td>AB</td>
<td>-</td>
<td>ER</td>
</tr>
<tr>
<td>18</td>
<td>45</td>
<td>M</td>
<td>O</td>
<td>+</td>
<td>SDS</td>
</tr>
<tr>
<td>19</td>
<td>55</td>
<td>M</td>
<td>O</td>
<td>+</td>
<td>Med/Surg</td>
</tr>
<tr>
<td>20</td>
<td>71</td>
<td>M</td>
<td>A</td>
<td>+</td>
<td>ER</td>
</tr>
</tbody>
</table>
Module 15 Two-way Tables (1) continued

Using two way tables to find percentages and probabilities

Directions: Use your two way tables created in Activity 1, to answer the following questions.

1. What percent of the patients were female?

2. What percent of the patients were male?

3. What percent of the patients had an age between 18-35 years old?

4. What percent of the patients were 65 years old or older?

5. What percent of patients had type O blood?

6. What percent of patients had type AB blood?

7. What percent of patients were having a Same Day Surgery?

8. What percent of patients were in Intensive Care?

9. What percent of patients had Rh+ blood?

10. What percent had Rh- blood?

11. Which do you think is more common, Rh+ or Rh-?

12. If a random person was chosen out of the group, what is the probability that they had a blood type of A?

13. If a random person was chosen out of the group, what is the probability that they had a blood type of B?

14. If a random person was chosen out of the group, what is the probability that the person was 36-49 years old?

15. If a random person was chosen out of the group, what is the probability that the person was 50-64 years old?

16. If a random person was chosen out of the group, what is the probability that the person went to the Emergency Room?

17. If a random person was chosen out of the group, what is the probability that the person went to the Medical/Surgical floor?
1. It is said the 11% of boys are left handed and 14 % of girls are left handed. Assume that 46 % of children are female. Use a convenient total for the number of children and fill out the following table using this information. Then use the table to answer the following questions.

<table>
<thead>
<tr>
<th></th>
<th>Left Handed</th>
<th>Right Handed</th>
<th>Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Girl</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Boy</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Totals</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

What is the probability of a right handed child being a boy?
What is the probability of a child being right handed and a boy?
What is the probability of a child being right handed or a boy?
What is the probability of a boy being right handed?

2. The following table describes the gender and majors of randomly selected students at a local college. Use the table to answer the following questions.

<table>
<thead>
<tr>
<th></th>
<th>Business</th>
<th>English</th>
<th>History</th>
<th>Music</th>
<th>Biology</th>
<th>Math</th>
</tr>
</thead>
<tbody>
<tr>
<td>Female</td>
<td>95</td>
<td>58</td>
<td>75</td>
<td>62</td>
<td>56</td>
<td>9</td>
</tr>
<tr>
<td>Male</td>
<td>102</td>
<td>64</td>
<td>59</td>
<td>53</td>
<td>78</td>
<td>14</td>
</tr>
</tbody>
</table>

a) Find all the row and column totals for the table.
b) What percent of the students are female History majors?
c) If we randomly select a student, what is the probability that the person is female and a Music major?
d) If we randomly select a student, what is the probability that the person is male and a History major?
e) If we randomly select a student, what is the probability that the person is female or a Math major?
f) What is the probability of a Math major being male?
g) What is the probability of a female student being a History major?
Module 15  Creating Tables from Percentages

Creating Tables from Percentage information

Directions: If we only know percent information, sometimes it is helpful to assume that we have 10,000 or 100,000 individuals, and then make a table from the percents. This table could then be used to find more complicated probabilities.

1. It is said the 9.5% of boys are left handed and 12.5% of girls are left handed. Assume that 48% of children are female. Assume there are 10,000 children, and fill out the following table using this information. Then use the table to answer the following question. If a child is right handed, what is the probability that the child is a boy?

<table>
<thead>
<tr>
<th></th>
<th>Left Handed</th>
<th>Right Handed</th>
<th>Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Girl</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Boy</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Totals</td>
<td></td>
<td></td>
<td>10000</td>
</tr>
</tbody>
</table>

\[ P(\text{Boy} \mid \text{Right Handed}) = ??? \]

2. Approximately 24% of the U.S. population smoke cigarettes. Approximately 10% of people who smoke cigarettes will develop lung cancer. If a person has never smoked cigarettes, they have approximately a 0.3% chance of getting lung cancer. Assume that we have 100,000 American adults. Fill out the following table. Then use the table to answer the following question. If a person has lung cancer, what is the probability that they smoke cigarettes?

<table>
<thead>
<tr>
<th></th>
<th>Smokes Cigarettes</th>
<th>Does not Smoke Cigarettes</th>
<th>Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gets Lung Cancer</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Does not get Lung Cancer</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Totals</td>
<td></td>
<td></td>
<td>100,000</td>
</tr>
</tbody>
</table>

\[ P(\text{Smokes Cigarettes} \mid \text{Has Lung Cancer}) = ??? \]
Module 15  Conditional Probability

Conditional Probabilities

Directions: The following table describes the gender and majors of 692 randomly selected students at a local college. Use the table to answer the following questions.

<table>
<thead>
<tr>
<th></th>
<th>Business</th>
<th>English</th>
<th>History</th>
<th>Music</th>
<th>Biology</th>
<th>Math</th>
</tr>
</thead>
<tbody>
<tr>
<td>Female</td>
<td>89</td>
<td>71</td>
<td>62</td>
<td>48</td>
<td>56</td>
<td>9</td>
</tr>
<tr>
<td>Male</td>
<td>112</td>
<td>58</td>
<td>59</td>
<td>53</td>
<td>62</td>
<td>13</td>
</tr>
</tbody>
</table>

1. Find all the row and column totals for the table.

2. If a person is a business major, what is the probability that the person is female?

3. If a person is male, what is the probability that the person is a biology major?

4. If we randomly select a Music major, what is the probability that the person is male?

5. What is the probability of choosing a person that is an English major, if we are given that the person is female?

6. What is the probability of choosing a male student if we are given that the person is a Math major.

7. If a student is a Math or Biology major, what is the probability that the person is female?

8. Of all the history majors, what percent are male?

9. Of all the female students, what percent are English majors?

10. Of all the male students, what percent are Business or History majors?
Module 15 Two-way Tables (3)

1. This data comes from a study of the factors that impact birth weight. Here the variable *Visit Doctor* indicates whether a woman visited a physician during the first trimester of her pregnancy. The variable *Low Weight* indicates whether a baby was born weighing less than 2500 grams.

<table>
<thead>
<tr>
<th>Visit Doctor</th>
<th>Low Weight</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No</td>
<td>Yes</td>
<td>Row Totals</td>
</tr>
<tr>
<td>Yes</td>
<td>66</td>
<td>23</td>
<td></td>
</tr>
<tr>
<td>No</td>
<td>64</td>
<td>36</td>
<td></td>
</tr>
<tr>
<td>Column Totals</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Complete the table.

Does visiting a doctor during the early stages of pregnancy seem to be associated with a lower incidence of low weight births? Identify the explanatory and response variables, and be sure to use math to support your conclusion and show your work.

If a woman does not visit the doctor during the first trimester of her pregnancy, how increased is the likelihood that she will have a low weight baby?

What is the probability of a woman visiting a doctor and having low birth weight?

What is the probability of a woman that has had a low birth weight having visited a doctor?

2. This table is based on records of accidents compiled by a State Highway Safety and Motor Vehicles Office (the marginal distributions and the lower right-hand corner have been filled in for you). Are people less likely to have a fatal accident if they are wearing a seatbelt? Be sure to clearly identify the explanatory and response variables, use math to support your conclusion, and show your work.

<table>
<thead>
<tr>
<th>Injury</th>
<th>Nonfatal Injury</th>
<th>Fatal Injury</th>
<th>Row Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seat belt</td>
<td>412,368</td>
<td>510</td>
<td>412,878</td>
</tr>
<tr>
<td>No seat belt</td>
<td>162,527</td>
<td>1,601</td>
<td>164,128</td>
</tr>
<tr>
<td>Column Total</td>
<td>574,895</td>
<td>2,111</td>
<td>577,006</td>
</tr>
</tbody>
</table>
3. A study in Sweden looked at the impact of playing soccer on the incidence of arthritis of the hip or knee. They gathered information on former elite soccer players, people who played soccer but not at the elite level, and those who never played soccer. Fill in the marginal distributions and the lower right-hand corner. Does this study suggest that playing soccer makes someone more likely to have arthritis of the hip or knee? Be sure to clearly identify the explanatory and response variables, use math to support your conclusion, and show your work.

<table>
<thead>
<tr>
<th>Soccer Player</th>
<th>Elite</th>
<th>Non-elite</th>
<th>Did not play</th>
<th>Column Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hip Arthritis</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Arthritis</td>
<td>10</td>
<td>9</td>
<td>24</td>
<td></td>
</tr>
<tr>
<td>No arthritis</td>
<td>61</td>
<td>206</td>
<td>548</td>
<td></td>
</tr>
<tr>
<td>Row Totals</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Four hundred seventy-eight students in grades 4 through 6 in selected schools in Michigan, were asked the following question.

Which of the following would make you popular among your friends? Rank in order.
Making good grades
Having lots of money
Being good at sports
Being handsome or pretty

4. The table below lists the number of students (by gender) who gave the indicated factor a ranking of 1 (most important factor in making them popular amongst their friends).

<table>
<thead>
<tr>
<th>Most Important Popularity Factor</th>
<th>Grades</th>
<th>Money</th>
<th>Sports</th>
<th>Looks</th>
<th>Row Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gender</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Girl</td>
<td>55</td>
<td>17</td>
<td>38</td>
<td>141</td>
<td></td>
</tr>
<tr>
<td>Boy</td>
<td>39</td>
<td>17</td>
<td>127</td>
<td>44</td>
<td></td>
</tr>
<tr>
<td>Column Totals</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

What proportion of the total number of students considers looks to be the most important factor in making them popular amongst their friends?

If we were to randomly select a student, what is the probability that the student would think Money is the most important factor in making them popular amongst their friends?
Are boys more likely to think that money is the most important factor in making them popular amongst their friends?

Suppose we randomly select a student. What is the probability that the student is a boy?

What is the probability that a student ranked grades as the most important factor in making him/her popular amongst his/her friends?

What is the probability that a boy ranked sports as the most important factor in making him popular amongst his friends?

What is the probability that a girl ranked looks as the most important factor in making her popular amongst her friends?

Suppose we randomly select a student. What is the probability that the student is a boy and the student ranked grades as the most important factor in making him popular amongst his friends?

Find and interpret the following probabilities and then indicate

i) \( P(\text{Money} | \text{Girl}) \)

ii) \( P(\text{Sports}) \)

iii) \( P(\text{Girl and Sports}) \)

This data is from a 5-year experiment with physicians between the ages of 40 and 84, published in 1988 by the Steering Committee of the Physicians Health Study Research Group. The physicians participating in the study were randomly selected to receive an aspirin or a placebo. The pills looked the same and the physicians did not know which they were taking.

<table>
<thead>
<tr>
<th></th>
<th>Heart attack</th>
<th>No heart attack</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Aspirin</strong></td>
<td>104</td>
<td>10,933</td>
</tr>
<tr>
<td><strong>Placebo</strong></td>
<td>189</td>
<td>10,845</td>
</tr>
</tbody>
</table>

Does this data suggest that taking aspirin reduces the risk of heart attack?
Module 16 Independent and Disjoint Variables

The following two way table gives the genders and majors of 114 randomly selected science students. (Note: None of these students were double majors.)

<table>
<thead>
<tr>
<th></th>
<th>Biology</th>
<th>Chemistry</th>
<th>Physics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Female</td>
<td>31</td>
<td>14</td>
<td>5</td>
</tr>
<tr>
<td>Male</td>
<td>33</td>
<td>19</td>
<td>12</td>
</tr>
</tbody>
</table>

1. a) Are Biology majors and Physics majors disjoint? Why?
   b) Find the probability of a person being a Biology major or a Physics major?
   c) Is the answer in part (b) the same as the probability of someone being a Biology major plus the probability of someone being a Physics major?

2. a) Are Chemistry's majors and males disjoint? Why?
   b) Find the probability of a person being a Chemistry major or male?
   c) Is the answer in part (b) the same as the probability of a Chemistry major plus the probability of someone being male?

3. a) Are the Physics majors and females independent of one another or dependent? Why? What probabilities could we use to show that our answer is correct?
   b) If we are given a person is female, what is the probability they are a Physics major? Is this the same as the probability that anyone is a physics major?
   c) If we are given that a person is a Physics major, what is the probability that they are female? Is that the same as the probability of anyone being female?
   d) Find the probability that a student is both female and a Physics major? Is this the same as the probability of someone being female times the probability of someone being a Physics major?
Module 16 Rules of Probability Worksheet

A striking trend in higher education is that more women than men reach each level of attainment. Here are the counts (in thousands) of earned degrees in the United States in the 2010-2011 academic year (classified by level and gender).

<table>
<thead>
<tr>
<th></th>
<th>Bachelor’s</th>
<th>Master’s</th>
<th>Professional</th>
<th>Doctorate</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Female</strong></td>
<td>986</td>
<td>411</td>
<td>52</td>
<td>32</td>
<td>1481</td>
</tr>
<tr>
<td><strong>Male</strong></td>
<td>693</td>
<td>260</td>
<td>45</td>
<td>27</td>
<td>1025</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>1679</td>
<td>671</td>
<td>97</td>
<td>59</td>
<td>2506</td>
</tr>
</tbody>
</table>

If you choose a degree recipient at random, what is the probability that the person you choose is a woman?

What is the conditional probability that you choose a woman, given that the person chosen received a doctorate?

Are the events “choose a woman” and “choose a doctoral degree recipient” independent? How do you know?

What is the probability that a randomly chosen degree recipient is a woman and the degree is a doctorate?

Use the College-Degree data from the previous problem to answer the following questions.

What is the probability that a randomly chosen degree recipient is a man?

What is the conditional probability that the person chosen received a bachelor’s degree, given that he is a man?

What is the probability that a randomly chosen degree recipient is a man and the degree conferred is a doctorate?

3) The probability that the team will win the play-off game is .7. What is the probability that the team will not win the play-off game?

4) There are 3 yellow cards, 5 blue cards, and 6 green cards. What is the probability that you pick a yellow or a blue card?
5) A piggy bank consists of 8 quarters, 15 dimes, 20 nickels, and 35 pennies. If you shake one coin out, find the following probabilities.

a) \( P(\text{nickel}) = \)

b) \( P(\text{dime or a nickel}) = \)

c) \( P(\text{quarter}) = \)

d) \( P(\text{not a penny}) = \)

6) There are purple, orange, yellow, and green gum drops in a bowl. If the probability of picking a purple is 26%, a yellow is 48%, and a green is 12%. What is the probability of picking an orange?

a) Using the probability from #6, is it more likely to pick an orange or purple or green or yellow? Justify your answer.

b) Give an example of a probability that would be equal to 0.

c) Give an example of a probability that would be equal to 1.

d) Can you have a probability that is \( \frac{7}{6} \)? Explain.
Module 16  Joint Probabilities

Directions: The following table describes the gender and majors of 692 randomly selected students at a local college. Use the table to answer the following questions.

<table>
<thead>
<tr>
<th></th>
<th>Business</th>
<th>English</th>
<th>History</th>
<th>Music</th>
<th>Biology</th>
<th>Math</th>
</tr>
</thead>
<tbody>
<tr>
<td>Female</td>
<td>89</td>
<td>71</td>
<td>62</td>
<td>48</td>
<td>56</td>
<td>9</td>
</tr>
<tr>
<td>Male</td>
<td>112</td>
<td>58</td>
<td>59</td>
<td>53</td>
<td>62</td>
<td>13</td>
</tr>
</tbody>
</table>

1. Find all the row and column totals for the table.

2. What percent of the students are female Music majors?

3. What percent of the students are male English majors?

4. If we randomly select a student, what is the probability that the person is female and a Math major?

5. If we randomly select a student, what is the probability that the person is male and a Biology major?

6. If we randomly select a student, what is the probability that the person is female or a Business major?

7. If we randomly select a student, what is the probability that the person is male or a History major?

8. If we randomly select a student, what is the probability that the person is female or an English major?

9. What percent of the students are either female or a music major?

10. What percent of the students are either male or a math major?

11. What is the probability of a English major being male?

12. What is he probability of a male student being an English major?

13. What is the probability of a student being male and an English major?
Section 16 Probability Distributions and Expected Values

A biology teacher assigns a large project on plants, but every semester, many of her students turn in the project late. In looking at her records she finds the following totals: Out of 400 total students that turned in a project, 342 turned it in on time, 21 turned it in one day late, 15 turned it in two days late, 11 turned it in three days late, 7 turned it in four days late, and 4 turned it in five days late.

1. Let X represent the random variable describing the number of days the project was late and P(x) represent the probability of a student turning in their project X days late. Fill out the following probability table with all the missing probabilities.

<table>
<thead>
<tr>
<th>X</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(X)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. From your probability distribution in #1, answer the following questions.

a) If a student does turn in their project late, how many days late is it most likely to be? Why?

b) What is the probability that a student turns in their project on time or one day late?

c) What is the probability that a student does not turn in their project on time? The teacher’s current biology class has 36 students. Estimate how many of the 36 students will not turn in their paper on time?

d) Find the expected value for the probability distribution. Write a sentence explaining the meaning of the expected value in this context.

A casino in Las Vegas offers the following gambling game. To play you must pay $5. You role a die one time. If you role a 1, 2 or 3, you lose your $5. If you role a 4 or a 5, you win $3. (You get your $5 back plus an additional $3). If you role a 6, you win $7. (You get your $5 back plus an additional $7.)

3. Let X represent the random variable describing the amount of money won or lost and P(x) represent the probability of winning or losing that money. Fill out the following probability table with all the missing probabilities.

<table>
<thead>
<tr>
<th>Money Lost or Won</th>
<th>-$5</th>
<th>+$3</th>
<th>+$7</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(x)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4. From your probability distribution in #3, answer the following questions.

a) If a person plays the dice game, what is the probability that they will win some money?

b) Find the expected value for the probability distribution. Write a sentence explaining the meaning of the expected value in this context.

c) Is this a fair game? Explain how you know?
The following are probability distributions for variable x. Answer the given questions for each:

1.

<table>
<thead>
<tr>
<th>x</th>
<th>2</th>
<th>3</th>
<th>9</th>
<th>11</th>
<th>12</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(x)</td>
<td>0.1</td>
<td>0.3</td>
<td>0.1</td>
<td>0.2</td>
<td>0.2</td>
<td></td>
</tr>
</tbody>
</table>

a) \( P(x \leq 9) \)
b) \( P(x = 20) \)
c) \( P(x \geq 3) \)
d) The mean for variable x

e) The standard deviation for variable x

2.

<table>
<thead>
<tr>
<th>x</th>
<th>5</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>13</th>
<th>15</th>
<th>20</th>
<th>21</th>
<th>23</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(x)</td>
<td>0.16</td>
<td>0.05</td>
<td>0.05</td>
<td>0.1</td>
<td>0.1</td>
<td>0.2</td>
<td>0.1</td>
<td>0.2</td>
<td></td>
</tr>
</tbody>
</table>

a) \( P(x > 10) \)
b) \( P(x \geq 10) \)
c) \( P(x < 20) \)
d) \( P(x = 8 \text{ or } x = 10) \)
e) The mean for variable x

f) The standard deviation for variable x.

3.

<table>
<thead>
<tr>
<th>x</th>
<th>2</th>
<th>6</th>
<th>10</th>
<th>11</th>
<th>16</th>
<th>20</th>
<th>25</th>
<th>30</th>
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</thead>
<tbody>
<tr>
<td>P(x)</td>
<td>0.16</td>
<td>0.05</td>
<td>0.05</td>
<td>0.1</td>
<td>0.2</td>
<td>0.1</td>
<td>0.1</td>
<td>0.2</td>
</tr>
</tbody>
</table>

a) \( P(x > 6) \)
b) \( P(x \geq 11) \)
c) \( P(x < 10) \)
d) \( P(x = 11 \text{ or } x = 10) \)
e) The mean for variable x

f) The standard deviation for variable x.
Section 17  Empirical Rule for Normal Distributions

Directions: Use the Empirical Rule (68-95-99.7) Rule for normal distributions to answer the following questions.

1. Consumer Reports Magazine wrote an article stating that monthly charges for cell phone plans in the U.S. are normally distributed with a mean of $62 and a standard deviation of $18.

Draw a picture of the normal curve with the cell phone charges for 1, 2 and 3 standard deviations above and below the mean.

What percent of people in the U.S. have a cell phone bill between $62 and $80 per month?
What are the two cell phone charges that the middle 95% of people are in between?
What percent of people in the U.S. have a monthly cell phone bill between $26 and $44.
Find the cell phone bill that 99.85% of people are less than.

2. The ACT exam is used by colleges across the country to make a decision about whether a student will be admitted to their college. ACT scores are normally distributed with a mean average of 21 and a standard deviation of 5.

a) Draw a picture of the normal curve with the ACT scores for 1, 2 and 3 standard deviations above and below the mean.

What percent of students score higher than a 31 on the ACT?
What are the two ACT scores that the middle 68% of people are in between?
What percent of people score between a 16 and 21 on the ACT?
Find the ACT score that 84% of people score less than?

3. Human pregnancies are normally distributed and last a mean average of 266 days and a standard deviation of 16 days.

Draw a picture of the normal curve with the pregnancy lengths for 1, 2 and 3 standard deviations above and below the mean.

What percent of pregnancies last between 218 days and 234 days?
Find two pregnancy lengths that the middle 68% of people are in between. This is the range of days that pregnancies typically take.
A marine came home from a long tour of duty and was amazed to hear that his wife was pregnant and expecting a baby. This was especially amazing since it had been 314 days since he had last seen his wife. His wife claims that the baby is just very late in coming. What is the probability that a pregnancy would last 314 days or more? What does this tell you about the wife’s claim?
Section 17  Finding and Interpreting z-scores

1. A sample of the heights of women had a normal distribution with a mean height of 63.9 inches and a standard deviation of 2.8 inches.

   a) One woman had a height of 67 inches. Find and interpret the z-score corresponding to this height. Find the probability of a woman being taller than 67 inches. Is she unusually tall compared to the rest of the women in the sample?

   b) One woman had a height of 57 inches. Find and interpret the z-score corresponding to this height. Find the probability of a woman being shorter than 57 inches. Is she unusually short compared to the rest of the women in the sample?

   c) One woman had a height of 73 inches. Find and interpret the z-score corresponding to this height. Find the probability of a woman being taller than 73 inches. Is she unusually tall compared to the rest of the women in the sample?

   d) One woman had a height of 59 inches and another woman had a height of 66 inches. Find the z-scores for both women. Which was more unusual?

2. IQ tests are normally distributed with a mean of 100 and a standard deviation of 15.

   a) A girl scored a 140 on the IQ test. Find and interpret the z-score for this IQ. Find the probability of someone scoring higher than 140. Is it unusual for someone to score a 140?

   b) A boy scored a 90 on the IQ test. Find the probability of someone scoring lower than 90. Find and interpret the z-score for this IQ. Is it unusual for someone to score a 90?

   c) One man scored a 120 on the IQ test. Find the probability of someone scoring higher than 120. Find and interpret the z-score for this IQ. Is it unusual for someone to score a 120?

   d) Mike scored a 135 on the IQ test and Jake scored a 71 on the IQ test. Find the Z-scores for each. Which score was more unusual?
Use a table for normal distributions to answer the following:

For a population the mean and the standard deviation are given to be: \( \mu = 14, \sigma = 3 \).
Find \( P(x > 16) \)

\[ P(x > 12) \]
\[ P(x < 17) \]
\[ P(x < 13) \]
\[ P(15 < x < 16) \]
\[ P(12 < x < 16) \]

For a population the mean and the standard deviation are given to be: \( \mu = 28, \sigma = 5 \).
a) Find \( P(x > 22) \)

\[ P(x > 32) \]
\[ P(x < 10) \]
\[ P(x > 8) \]
\[ P(26 < x < 28) \]
\[ P(29 < x < 30) \]

For a population the mean and the standard deviation are given to be: \( \mu = 12, \sigma = 4 \).
Find \( P(x < 10) \)

\[ P(x > 11) \]
\[ P(x > 14) \]
\[ P(x > 6) \]
\[ P(6 < x < 10) \]
\[ P(10 < x < 14) \]
Module 17 Normal Distribution activity (2)

Use a table for normal distributions to answer the following:

For a population the mean and the standard deviation are given to be: \( \mu = 20, \sigma = 1.5 \).

1. Find \( P(x > 18) \)
2. \( P(x > 22) \)
3. \( P(x < 19) \)
4. \( P(x < 15) \)
5. \( P(19 < x < 23) \)
6. \( P(19 < x < 20) \)

For a population the mean and the standard deviation are given to be: \( \mu = 26, \sigma = 4 \).

7. Find \( P(x > 24) \)
8. \( P(x > 27) \)
9. \( P(x < 27) \)
10. \( P(x > 15) \)
11. \( P(26 < x < 28) \)
12. \( P(25 < x < 30) \)

For a population the mean and the standard deviation are given to be: \( \mu = 12, \sigma = 4 \).

13. Find \( P(x < 11) \)
14. \( P(x > 11) \)
15. \( P(x > 16) \)
16. \( P(x > 8) \)
17. \( P(9 < x < 11) \)
## Z-Table (area to the left of Z)

<table>
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<th>0.01</th>
<th>0.02</th>
<th>0.03</th>
<th>0.04</th>
<th>0.05</th>
<th>0.06</th>
<th>0.07</th>
<th>0.08</th>
<th>0.09</th>
</tr>
</thead>
<tbody>
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<td>0.003</td>
<td>0.003</td>
<td>0.003</td>
<td>0.003</td>
<td>0.003</td>
<td>0.003</td>
<td>0.003</td>
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<td>0.002</td>
</tr>
<tr>
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<td>0.005</td>
<td>0.005</td>
<td>0.004</td>
<td>0.004</td>
<td>0.004</td>
<td>0.004</td>
<td>0.004</td>
<td>0.004</td>
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</tr>
<tr>
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<td>0.005</td>
<td>0.005</td>
<td>0.005</td>
</tr>
<tr>
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<td>0.007</td>
<td>0.007</td>
</tr>
<tr>
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<td>0.013</td>
<td>0.013</td>
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<td>0.012</td>
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<td>0.011</td>
<td>0.011</td>
<td>0.010</td>
<td>0.010</td>
</tr>
<tr>
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<td>0.018</td>
<td>0.018</td>
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<td>0.015</td>
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<td>0.014</td>
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<tr>
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<td>0.025</td>
<td>0.024</td>
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<td>0.023</td>
<td>0.022</td>
<td>0.021</td>
<td>0.021</td>
<td>0.020</td>
<td>0.019</td>
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<tr>
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<td>0.034</td>
<td>0.033</td>
<td>0.032</td>
<td>0.031</td>
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<td>0.029</td>
<td>0.028</td>
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<tr>
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<td>0.042</td>
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<td>0.039</td>
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</tr>
<tr>
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<td>0.062</td>
<td>0.060</td>
<td>0.059</td>
<td>0.057</td>
<td>0.055</td>
<td>0.054</td>
<td>0.052</td>
<td>0.051</td>
<td>0.049</td>
<td>0.048</td>
</tr>
<tr>
<td>-2.4</td>
<td>0.082</td>
<td>0.080</td>
<td>0.078</td>
<td>0.076</td>
<td>0.075</td>
<td>0.073</td>
<td>0.071</td>
<td>0.069</td>
<td>0.068</td>
<td>0.066</td>
</tr>
<tr>
<td>-2.3</td>
<td>0.107</td>
<td>0.104</td>
<td>0.102</td>
<td>0.099</td>
<td>0.096</td>
<td>0.094</td>
<td>0.091</td>
<td>0.089</td>
<td>0.087</td>
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<td>0.122</td>
<td>0.119</td>
<td>0.116</td>
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<td>0.110</td>
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<tr>
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<td>0.162</td>
<td>0.158</td>
<td>0.154</td>
<td>0.150</td>
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<td>0.143</td>
</tr>
<tr>
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<td>0.217</td>
<td>0.212</td>
<td>0.207</td>
<td>0.202</td>
<td>0.197</td>
<td>0.192</td>
<td>0.188</td>
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</tr>
<tr>
<td>-1.9</td>
<td>0.287</td>
<td>0.281</td>
<td>0.274</td>
<td>0.268</td>
<td>0.262</td>
<td>0.256</td>
<td>0.250</td>
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<td>0.239</td>
<td>0.233</td>
</tr>
<tr>
<td>-1.8</td>
<td>0.359</td>
<td>0.351</td>
<td>0.344</td>
<td>0.336</td>
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<tr>
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<td>0.446</td>
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<tr>
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