Topics to be Included:

Separable Equations: \( \frac{dy}{dx} = f(x)g(y) \)  
- Reconfigure by writing: \( \frac{dy}{g(y)} = f(x)dx \), then integrate both sides, and solve for \( y(x) \)

Linear Equations: \( y' + p(x)y = g(x) \)  
- Multiply the integrating factor \( \mu = e^{\int p(x)dx} \) on both sides of the above equation, then rewrite it as \( (\mu y)' = g(x) \), and solve for \( y \): \( y = \frac{\int g(x)dx}{\mu(x)} + C \)

Exact Equations: \( M(x, y) + N(x, y)y' = 0 \)  
- First check that \( M_y = N_x \). If this holds, then the equation is exact provided that \( M(x, y) \) and \( N(x, y) \) have continuous partial derivatives on some square interval.
- To solve, let \( \psi_x = M, and \psi_y = N \). Now find \( \psi(x, y) = \int \psi_x dx = \int M(x, y)dx + h(y) \). Now use the fact that \( \frac{\partial}{\partial y} \psi(x, y) = N(x, y) \) to find \( h(y) \).
- The answer is \( \psi(x, y) = C \) and \( C \) can be found if an initial condition is set, and \( y \) can be explicitly solved within the implicit expression \( \psi(x, y) = C \).

Substitution Methods: (see section 1.6)

- "Linear-looking" Equations in the form \( \frac{dy}{dx} = F(ax + by + c) \)
  - Let \( v = ax + by + c \), then \( \frac{dv}{dx} = a + b \frac{dy}{dx} \). Therefore, \( \frac{dy}{dx} = \frac{1}{b} \left( \frac{dv}{dx} - a \right) \)
    - Now make the substitution, solve for \( v(x) \), and then solve for \( y(x) \). See Example 3 on p. 73 of section 2.5

- Homogeneous Equations
  - These either have the form \( \frac{dy}{dx} = F \left( \frac{y}{x} \right) \) or have the more complex form \( Ax^m y^n \frac{dy}{dx} = Bx^p y^q + Cx^r y^s \) where \( m + n = p + q = r + s = K \). Divide the latter equation by \( x^K \) and place it in the form of the former equation. Then make the substitution \( v = \frac{y}{x}, y = vx, \frac{dy}{dx} = v + x \frac{dv}{dx} \)
    - See example 1 of section 2.5, and number 1-10 of exercise 2.5

- Bernoulli Equations: \( \frac{dy}{dx} + p(x)y = q(x)y^n \)
  - Make the substitution \( v = y^{1-n} \), which will transform the above equation into \( \frac{dv}{dx} + (1-n)p(x)v = (1-n)q(x) \), which is a linear equation, and can be solved with the integrating factor \( \mu = e^{\int (1-n)p(x)dx} \)
    - See example 2 of section 2.5, and 15-20 of section 2.5
Setting Up and Solving Models

- Growth and Decay with Carbon Dating
  - These primarily involve the basic model of \( P(x) = P_0 e^{kx} \) where \( k \) is the growth or decay constant, and \( P_0 \) is the initial value. See #1-12 of section 3.1, and #1-4 of section 1.3

- Newton’s Law of Cooling/Warming
  - See #5-6 of section 1.3, and #13-20 of section 3.1

- Mixtures
  - #9-12 of section 1.3, #21-26 of section 3.1, #5-8 of section 3.3

Theoretical Questions

- Be sure you understand the notion of the slope/directions field, and its relationship to first order differential equations
- Be sure to understand theorem 1.2.1 of section 1.2 concerning the existence and uniqueness of a first order IVP.
- Be sure to understand the definition of autonomous differential equations and be able to sketch their one-dimensional phase portrait, identifying attractors, repellers, and a possible solution.
- Be sure that you can solve an IVP entirely up to the point of identifying its explicit solution, complete with its domain of definition. An example of this may be found in Paul’s Notes on Differential Equations -> Exact Equations -> Example 2:

  http://tutorial.math.lamar.edu/Classes/DE/Exact.aspx