1. Interpret the gradient field of the temperature function $T = f(x,y)$.

Choose the correct answer below.

- $A$. It shows the time at which the temperature begins to increase and the time when the temperature begins to decrease.
- $B$. It shows the direction in which the temperature increases the fastest and the amount of increase.
- $C$. It shows the direction in which the temperature decreases the fastest and the amount of decrease.
- $D$. It shows the temperature increase for a specific time.

2. Make a sketch of the following vector field.

$$ F = \langle 2y, 2x \rangle $$

Choose the correct vector field below.

- $A$.
- $B$.
- $C$.
- $D$.

3. Make a sketch of the following vector field.

$$ F = \langle 3e^{-x}, 0 \rangle $$

Choose the correct vector field below.

- $A$.
- $B$.
- $C$.
- $D$. 
4. Determine whether the vector field \( \mathbf{F} \) is tangent to or normal to the curve \( C \) at points on \( C \). A vector \( \mathbf{n} \) normal to \( C \) is also given. Sketch \( C \) and a few representative vectors of \( \mathbf{F} \).

\[
\mathbf{F} = \langle x, y \rangle, \quad \text{where} \quad C = \{(x,y): x = -4\} \quad \text{and} \quad \mathbf{n} = \langle -4,0 \rangle
\]

Determine whether the vector field \( \mathbf{F} \) is tangent to or normal to \( C \) at points on \( C \). Choose the correct answer below.

\( \bigcirc \text{A.} \quad \text{The vector field } \mathbf{F} \text{ is normal to } C \text{ at } (-4,0). \)

\( \bigcirc \text{B.} \quad \text{The vector field } \mathbf{F} \text{ is tangent to } C \text{ at } (-4,0). \)

\( \bigcirc \text{C.} \quad \text{The vector field } \mathbf{F} \text{ is normal to } C \text{ at all points on } C. \)

\( \bigcirc \text{D.} \quad \text{The vector field } \mathbf{F} \text{ is tangent to } C \text{ at all points on } C. \)

Sketch \( C \) and a few representative vectors of \( \mathbf{F} \). Choose the correct graph below.

\( \bigcirc \text{A.} \)
\( \bigcirc \text{B.} \)
\( \bigcirc \text{C.} \)
\( \bigcirc \text{D.} \)
5. The gravitational force on a point mass \( m \) due to a point mass \( M \) is a gradient field with potential \( U(r) = \frac{GMm}{r} \), where \( G \) is the gravitational constant and \( r = \sqrt{x^2 + y^2 + z^2} \) is the distance between the masses. Answer parts \( a \) through \( e \).

**a.** Find the components of the gravitational force in the \( x-, y-, \) and \( z- \) directions, where \( \mathbf{F}(x,y,z) = -\nabla U(x,y,z) \). Choose the correct answer below.

- **A.** \( \mathbf{F}(x,y,z) = \frac{GMm}{(x^2 + y^2 + z^2)^{3/2}} \langle x, y, z \rangle \)
- **B.** \( \mathbf{F}(x,y,z) = \frac{GMm}{\sqrt{x^2 + y^2 + z^2}} \langle x, y, z \rangle \)
- **C.** \( \mathbf{F}(x,y,z) = \frac{GMm}{x^2 + y^2 + z^2} \langle x, y, z \rangle \)
- **D.** \( \mathbf{F}(x,y,z) = \frac{GMm}{(x^2 + y^2 + z^2)^2} \langle x, y, z \rangle \)

**b.** Show that the gravitational force points in the radial direction (outward from point mass \( M \)) and the radial component is \( F(r) = \frac{GMm}{r^2} \). Rewrite \( \mathbf{F} \) in terms of \( r \). Choose the correct answer below.

- **A.** \( F(r) = \frac{GMm}{|r|^2} r \)
- **B.** \( F(r) = \frac{GMm}{|r|^3} r \)
- **C.** \( F(r) = \frac{GMm}{|r|} \)
- **D.** \( F(r) = \frac{GMm}{|r|^3} \)

Rewrite \( \mathbf{F} \) again. Choose the correct answer below.
5. (cont.)

**A.** \( F(r) = \frac{GMm}{|r|} \)

**B.** \( F(r) = \frac{GMm}{|r|^2} \frac{r}{|r|^2} \)

**C.** \( F(r) = \frac{GMm}{|r|^2} \frac{r}{|r|} \)

**D.** \( F(r) = \frac{GMm}{|r|^2} \frac{1}{|r|} \)

**c.** Show that the vector field is orthogonal to the equipotential surfaces at all points in the domain of \( U \). Plot the equipotential surfaces of \( U \). What are the resulting shapes?

- **Spheres**
- **Hyperboloids**
- **Cylindroids**
- **Paraboloids**

What is a vector \( t \) that is always tangent to this shape?

- **A.** \( \langle -y, z, -x \rangle \)
- **B.** \( \langle xy, yz, 2xz \rangle \)
- **C.** \( \langle x, y, z \rangle \)
- **D.** \( \langle -yz, -xz, 2xy \rangle \)

Find the dot product of \( F \) and \( t \). What is the result?

☑️ (Simplify your answer.)

Therefore, by definition, the vector field is orthogonal to the equipotential curves at all points in the domain of \( U \).

6. Evaluate the following line integral.

\[
\int_{C} xy \, ds; \text{ C is the portion of the unit circle } \mathbf{r}(s) = \langle \cos s, \sin s \rangle, \text{ for } 0 \leq s \leq \frac{\pi}{2}
\]

The value of the line integral is ☐. (Simplify your answer.)
7. Use a scalar line integral to find the length of the following curve.

\[ r(t) = \left( 28 \sin \frac{t}{7}, 28 \cos \frac{t}{7}, \frac{t}{2} \right), \text{ for } 0 \leq t \leq 2 \]

The length of the curve is \[ \square \].

(Type an exact answer, using radicals as needed.)

8. Given the following vector field and oriented curve C, evaluate \( \int_C F \cdot T \, ds \).

\[ F = \langle y, x \rangle \] on the line segment from (2,3) to (6,10)

\[ \int_C F \cdot T \, ds = \square \]

(Type an exact answer, using radicals as needed.)

9. The figure to the right shows a plot of a vector field \( F \) along with three curves, \( C_1, C_2, \) and \( C_3 \). Determine whether each line integral \( \int_{C_i} F \cdot dr \), \( i = 1, 2, 3 \), is positive, negative, or zero.

The line integral \( \int_{C_1} F \cdot dr \) is \( \square \).

The line integral \( \int_{C_2} F \cdot dr \) is \( \square \).

The line integral \( \int_{C_3} F \cdot dr \) is \( \square \).
10. Consider the rotation field \( \mathbf{F} = \langle -y, x \rangle \) and the three paths shown in the figure. Compute the work done on each of the three paths. Does it appear that the line integral \( \int_C \mathbf{F} \cdot \mathbf{T} \, ds \) is independent of the path, where \( C \) is a path from (1,0) to (0,1)?

What is the value of the line integral for the path \( C_1 \)?

☐ (Type an exact answer, using \( \pi \) as needed.)

What is the value of the line integral for the path \( C_2 \), which is a quarter of the unit circle centered at the origin?

☐ (Type an exact answer, using \( \pi \) as needed.)

What is the value of the line integral for the path \( C_3 \)?

☐ (Type an exact answer, using \( \pi \) as needed.)

Does it appear that the line integral is independent of the path?

☐ No

☐ Yes

11. How do you determine whether a vector field in \( \mathbb{R}^2 \) is conservative (has a potential function \( \varphi \) such that \( \mathbf{F} = \nabla \varphi \))? Assume \( \mathbf{F} = \langle f, g \rangle \) is a vector field defined on a connected and simply connected region \( D \) of \( \mathbb{R}^2 \), where \( f \) and \( g \) have continuous first partial derivatives on \( D \). Choose the correct answer below.

☐ A. The condition \( \frac{\partial f}{\partial y} = \frac{\partial g}{\partial x} \) must be met in order for \( \mathbf{F} \) to be conservative.

☐ B. The condition \( \frac{\partial f}{\partial x} + \frac{\partial g}{\partial y} = 0 \) must be met in order for \( \mathbf{F} \) to be conservative.

☐ C. The condition \( \frac{\partial f}{\partial y} + \frac{\partial g}{\partial x} = 0 \) must be met in order for \( \mathbf{F} \) to be conservative.

☐ D. The condition \( \frac{\partial f}{\partial x} = \frac{\partial g}{\partial y} \) must be met in order for \( \mathbf{F} \) to be conservative.
12. If $\mathbf{F}$ is a conservative vector field on a region $\mathbf{R}$, what is the value of $\oint_C \mathbf{F} \cdot d\mathbf{r}$, where $C$ is a simple closed smooth oriented curve in $\mathbf{R}$.

Choose the correct answer below.

- **A.** The value of $\oint_C \mathbf{F} \cdot d\mathbf{r}$ is $-1$.
- **B.** The value of $\oint_C \mathbf{F} \cdot d\mathbf{r}$ is zero.
- **C.** The value of $\oint_C \mathbf{F} \cdot d\mathbf{r}$ is $1$.
- **D.** There is not enough information to determine the value of $\oint_C \mathbf{F} \cdot d\mathbf{r}$.

13. Determine whether the following vector field is conservative on $\mathbf{R}^2$. If so, determine the potential function.

$$\mathbf{F} = \langle 4x, 5y \rangle$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** $\mathbf{F}$ is conservative on $\mathbf{R}^2$. The potential function is $\varphi(x,y) = \boxed{\quad}$.
  (Use $C$ as the arbitrary constant.)
- **B.** $\mathbf{F}$ is not conservative on $\mathbf{R}^2$.

14. Determine whether the following vector field is conservative on $\mathbf{R}^3$. If so, determine the potential function.

$$\mathbf{F} = \langle 4y + z, 4x + 4z, x + 4y \rangle$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** $\mathbf{F}$ is conservative on $\mathbf{R}^3$. The potential function is $\varphi(x,y,z) = \boxed{\quad}$.
  (Use $C$ as the arbitrary constant.)
- **B.** $\mathbf{F}$ is not conservative on $\mathbf{R}^3$. 

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15. Evaluate the line integral \( \int_C \nabla \varphi \cdot \mathbf{dr} \) for the following function \( \varphi \) and oriented curve \( C \) in two ways.

(a) Use a parametric description of \( C \) and evaluate the integral directly.
(b) Use the Fundamental Theorem of line integrals.

\[ \varphi(x,y) = 5x + 6y; \quad C: \mathbf{r}(t) = \langle 2 - t, t \rangle, \text{ for } 0 \leq t \leq 2 \]

(a) Using a parametric description of \( C \), \( \int_C \nabla \varphi \cdot \mathbf{dr} = \square \).

(Type an exact answer, using radicals as needed.)

(b) Using the fundamental theorem of line integrals, \( \int_C \nabla \varphi \cdot \mathbf{dr} = \square \).

(Type an exact answer, using radicals as needed.)

16. Evaluate \( \int_C \nabla \left( e^{-y} \cos x \right) \cdot \mathbf{dr} \), where \( C \) is the line from \((0,0)\) to \((\pi, \ln 2)\), using any method.

\[ \int_C \nabla \left( e^{-y} \cos x \right) \cdot \mathbf{dr} = \square \, \text{ (Simplify your answer.)} \]

17. (a) For what values of \( a \), \( b \), \( c \), and \( d \) is the field \( \mathbf{F} = \langle ax + by, cx + dy \rangle \) conservative?
(b) For what values of \( a \), \( b \), and \( c \) is the field \( \mathbf{F} = \langle ax^2 - by^2, cxy \rangle \) conservative?

(a) \( \mathbf{F} = \langle ax + by, cx + dy \rangle \) is conservative for all values of \( a \), \( b \), \( c \), and \( d \) satisfying the condition(s) \( \square \).

(Type an equation. Use a comma to separate answers as needed.)

(b) \( \mathbf{F} = \langle ax^2 - by^2, cxy \rangle \) is conservative for all values of \( a \), \( b \), and \( c \) satisfying the condition(s) \( \square \).

(Type an equation. Use a comma to separate answers as needed.)
18. Consider the following region $R$ and the vector field $\mathbf{F}$.

a. Compute the two-dimensional curl of the vector field.
b. Evaluate both integrals in Green's Theorem and check for consistency.
c. State whether the vector field is conservative.

$\mathbf{F} = (4y, -4x)$; $R$ is the region bounded by $y = \sin x$ and $y = 0$, for $0 \leq x \leq \pi$.

a. The two-dimensional curl is $\square$.

b. $\oint \mathbf{F} \cdot d\mathbf{r} = \square$

c. Is the vector field conservative?

☐ No
☐ Yes
A two-dimensional vector field describes ideal flow if it has both zero curl and zero divergence on a simply connected region.

a. Verify that the curl and divergence of the given field is zero.

b. Find a potential function \( \varphi \) and a stream function \( \psi \) for the field.

c. Verify that \( \varphi \) and \( \psi \) satisfy Laplace's equation \( \varphi_{xx} + \varphi_{yy} = \psi_{xx} + \psi_{yy} = 0 \).

\[ \mathbf{F} = \langle 3x^3 - 9xy^2, 3y^3 - 9x^2y \rangle \]

a. Verify that the given vector field has zero curl.

- \( \bigcirc A. \) The difference of the partial derivatives, \( f_x \) and \( g_y \), equals zero.
- \( \bigcirc B. \) The difference of the partial derivatives, \( g_x \) and \( f_y \), equals zero.
- \( \bigcirc C. \) The sum of the partial derivatives, \( f_x \) and \( g_y \), equals zero.
- \( \bigcirc D. \) The sum of the partial derivatives, \( g_x \) and \( f_y \), equals zero.

Verify that the given vector field has zero divergence.

- \( \bigcirc A. \) The sum of the partial derivatives, \( g_x \) and \( f_y \), equals zero.
- \( \bigcirc B. \) The difference of the partial derivatives, \( f_x \) and \( g_y \), equals zero.
- \( \bigcirc C. \) The sum of the partial derivatives, \( f_x \) and \( g_y \), equals zero.
- \( \bigcirc D. \) The difference of the partial derivatives, \( g_x \) and \( f_y \), equals zero.

b. Find a potential function \( \varphi \) for \( \mathbf{F} \) where the arbitrary constant is \( C = 0 \).

\[ \varphi(x,y) = \square \]

Find a stream function \( \psi \) for \( \mathbf{F} \) where the arbitrary constant is \( C = 0 \).

\[ \psi(x,y) = \square \]

c. Verify that \( \varphi \) satisfies Laplace's equation.

\[ \varphi_{xx} + \varphi_{yy} = \square + \square = 0 \]

(Type the terms of your expression in the same order as they appear in the original expression.)

Verify that \( \psi \) satisfies Laplace's equation.

\[ \psi_{xx} + \psi_{yy} = \square + \square = 0 \]

(Type the terms of your expression in the same order as they appear in the original expression.)
20. Find the divergence of the following vector field.

\[ \mathbf{F} = \langle 8x, 5y, -6z \rangle \]

The divergence of \( \mathbf{F} \) is \( \square \).

21. Compute the curl of the following vector field.

\[ \mathbf{F} = \langle 4x^2 - y^2, xy, z \rangle \]

The curl of \( \mathbf{F} \) is \( \square \)i + \( \square \)j + \( \square \)k.

22. Is the following expression defined? If so, state whether the result is a scalar or a vector. Assume \( \mathbf{F} \) is a sufficiently differentiable vector field and \( \phi \) is a sufficiently differentiable scalar-valued function.

\[ \nabla \cdot \phi \]

Choose the correct answer below.

- A. Yes, vector field
- B. No
- C. Yes, scalar function

23. Give a parametric description of the form \( \mathbf{r}(u,v) = \langle x(u,v), y(u,v), z(u,v) \rangle \) for the frustum of the cone \( z^2 = x^2 + y^2 \), for \( 3 \leq z \leq 5 \).

Select the correct choice below.

- A. \( \mathbf{r}(u,v) = \left\langle \frac{3v}{5} \cos u, \frac{3v}{5} \sin u, v \right\rangle, 0 \leq u \leq 2\pi, 3 \leq v \leq 5 \)
- B. \( \mathbf{r}(u,v) = \langle 5v \cos u, 3v \sin u, v \rangle, 0 \leq u \leq 2\pi, 3 \leq v \leq 5 \)
- C. \( \mathbf{r}(u,v) = \langle 5v \cos u, 3v \sin u, v \rangle, 0 \leq u \leq 2\pi, 3 \leq v \leq 5 \)
- D. \( \mathbf{r}(u,v) = \langle v \cos u, v \sin u, v \rangle, 0 \leq u \leq 2\pi, 3 \leq v \leq 5 \)
- E. \( \mathbf{r}(u,v) = \left\langle \frac{5v}{3} \cos u, \frac{5v}{3} \sin u, v \right\rangle, 0 \leq u \leq 2\pi, 3 \leq v \leq 5 \)

24. Find the area of the plane \( z = 19 - 3x - 2y \) above the square \(|x| \leq 1, |y| \leq 1\) using a parametric description of the surface.

The area is \( \square \).

(Type an exact answer, using radicals as needed.)
25. Evaluate the surface integral \( \iint_{S} f(x,y,z) \, dS \) using an explicit representation of the surface.

\[
f(x,y,z) = 5xy, \text{ S is the plane } z = 3 - x - y \text{ in the first octant.}
\]

The value of the surface integral is \( \underline{\text{}} \).

(Type an exact answer, using radicals as needed.)

26. Find the flux of the vector field \( \mathbf{F} = \langle e^{-x}, 3z, 2xy \rangle \) across the curved sides of the cylinder \( S = \{(x,y,z): z = \cos y, \, |y| \leq \pi, \, 0 \leq x \leq 4\} \) with the normal vectors pointing upward. You may use either an explicit or parametric description of the surface.

The flux is \( \underline{\text{}} \).

(Simplify your answer.)
27. Let \( y = f(x) \) be a curve in the xy-plane with \( f(x) \neq 0 \), for \( a \leq x \leq b \). Let \( S \) be the surface generated when the graph of \( f \) on \([a,b]\) is revolved about the x-axis.

a. Show that \( S \) is described parametrically by \( \mathbf{r}(u,v) = (u, f(u) \cos v, f(u) \sin v) \), for \( a \leq u \leq b \), \( 0 \leq v \leq 2\pi \).

b. Find the integral that gives the surface area of \( S \).

c. Apply the result of part (b) to the surface generated with \( f(x) = x^3 \), for \( 0 \leq x \leq 1 \).

d. Apply the result of part (b) to the surface generated with \( f(x) = (4 - x^2)^{1/2} \), for \( 0 \leq x \leq 1 \).

\[
\text{a. To define } S \text{ parametrically, let } u = \quad \text{and let } v \text{ be the angle between the } \quad \text{and the function as it is being rotated about the x-axis.}
\]

b. Find the integral that gives the surface area of \( S \). Choose the correct answer below.

\[
\begin{align*}
\bigcirc A. & \quad \int_0^{2\pi} \int_a^b (f(u))^2 \sqrt{f'(u)^2 + 1} \, du \, dv \\
\bigcirc B. & \quad \int_0^{2\pi} \int_a^b \frac{(f(u))^2}{\sqrt{(f'(u))^2 + 1}} \, du \, dv \\
\bigcirc C. & \quad \int_0^{2\pi} \int_a^b f(u) \sqrt{(f'(u))^2 + 1} \, du \, dv \\
\bigcirc D. & \quad \int_0^{2\pi} \int_a^b f(u) \sqrt{f'(u) + 1} \, du \, dv
\end{align*}
\]

c. The surface area is \( \square \).
(Round to four decimal places as needed.)

d. The surface area is \( \square \).
(Type an exact answer, using \( \pi \) as needed.)
28. Why does a conservative vector field produce zero circulation around a closed curve?

Choose the correct answer below.

- **A.** A conservative vector field \( \mathbf{F} \) on a domain \( D \) has a potential function \( \varphi \) such that \( \mathbf{F} = \nabla \times \nabla \varphi \).
  
  Since \( \nabla \times \nabla \varphi = 0 \), it follows that \( \mathbf{F} = 0 \), and so the circulation integral \( \oint_{C} \mathbf{F} \cdot \, d\mathbf{r} = \iint_{S} (\nabla \times \mathbf{F}) \cdot \mathbf{n} \, dS \) is zero on all closed curves in \( D \).

- **B.** A conservative vector field \( \mathbf{F} \) on a domain \( D \) has a potential function \( \varphi \) such that \( \mathbf{F} = \nabla \varphi \). Since \( \nabla \times \nabla \varphi = \mathbf{n} \), it follows that \( \nabla \times \mathbf{F} = \mathbf{n} \), and so the circulation integral \( \oint_{C} \mathbf{F} \cdot \, d\mathbf{r} = \iint_{S} (\nabla \times \mathbf{F}) \cdot \mathbf{n} \, dS \) is zero on all closed curves in \( D \).

- **C.** A conservative vector field \( \mathbf{F} \) on a domain \( D \) has a potential function \( \varphi \) such that \( \mathbf{F} = \nabla \varphi \). Since \( \nabla \times \nabla \varphi = 0 \), it follows that \( \nabla \times \mathbf{F} = 0 \), and so the circulation integral \( \oint_{C} \mathbf{F} \cdot \, d\mathbf{r} = \iint_{S} (\nabla \times \mathbf{F}) \cdot \mathbf{n} \, dS \) is zero on all closed curves in \( D \).

29. Verify that the line integral and the surface integral of Stokes' Theorem are equal for the following vector field, surface \( S \), and closed curve \( C \). Assume that \( C \) has counterclockwise orientation and \( S \) has a consistent orientation.

\( \mathbf{F} = \langle x, y, z \rangle \); \( S \) is the paraboloid \( z = 5 - x^2 - y^2 \), for \( 0 \leq z \leq 5 \) and \( C \) is the circle \( x^2 + y^2 = 5 \) in the xy-plane.

Evaluate the line integral of Stokes' Theorem.

[ ] (Type an exact answer, using \( \pi \) as needed.)

Evaluate the surface integral of Stokes' Theorem.

[ ] (Type an exact answer, using \( \pi \) as needed.)

30. Evaluate the line integral \( \oint_{C} \mathbf{F} \cdot \, d\mathbf{r} \) by evaluating the surface integral in Stokes' Theorem with an appropriate choice of \( S \). Assume that \( C \) has a counterclockwise orientation.

\( \mathbf{F} = \langle 8xy \sin z, 4x^2 \sin z, 4x^2y \cos z \rangle \); \( C \) is the boundary of the plane \( z = 10 - 2x - 5y \) in the first octant.

\( \oint_{C} \mathbf{F} \cdot \, d\mathbf{r} = \) [ ] (Type an exact answer, using \( \pi \) as needed.)
31. For the following velocity field, compute the curl, make a sketch of the curl, and interpret the curl.

\[ \mathbf{v} = (1 - z^2, 0, 0) \]

The curl is \( \begin{bmatrix} \Box \end{bmatrix} \).

Make a sketch of the curl. Choose the correct graph below.

Interpret the curl. Choose the correct answer below.

- **A.** A paddle wheel with its axis oriented in the x-direction gives the maximum angular speed of the wheel. At \( y = 0 \), the speed of the paddle wheel is 0, but increases as values of \( y \) become larger (either positive or negative). The rotation of the paddle wheel (either clockwise or counterclockwise) depends on the sign of \( y \).

- **B.** A paddle wheel with its axis oriented in the z-direction gives the maximum angular speed of the wheel. At \( x = 0 \), the speed of the paddle wheel is 0, but increases as values of \( x \) become larger (either positive or negative). The rotation of the paddle wheel (either clockwise or counterclockwise) depends on the sign of \( x \).

- **C.** A paddle wheel with its axis oriented in the y-direction gives the maximum angular speed of the wheel. At \( z = 0 \), the speed of the paddle wheel is 0, but increases as values of \( z \) become larger (either positive or negative). The rotation of the paddle wheel (either clockwise or counterclockwise) depends on the sign of \( z \).

32. Suppose \( \text{div} \; \mathbf{F} = 0 \) in a region enclosed by two concentric spheres. What is the relationship between the outward fluxes across the two spheres?

Choose the correct answer below.

- **A.** The outward fluxes are opposite of each other.

- **B.** One outward flux equals double the other one.

- **C.** The outward fluxes are equal to each other.

- **D.** There is no relationship between the outward fluxes.
33. Evaluate both integrals of the Divergence Theorem for the following vector field and region.

\[ \mathbf{F} = \langle 2x, 4y, 3z \rangle; \quad D = \{ (x,y,z): x^2 + y^2 + z^2 \leq 4 \} \]

\[ \iiint_D \nabla \cdot \mathbf{F} \, dV = \square \]
(Type an exact answer, using \( \pi \) as needed.)

\[ \iint_S \mathbf{F} \cdot \mathbf{n} \, dS = \square \]
(Type an exact answer, using \( \pi \) as needed.)

34. Find the net outward flux of the field \( \mathbf{F} = \langle z, 9z - 4x, y - 9x \rangle \) across the sphere of radius 8 centered at the origin.

The net outward flux across the sphere is \( \square \).
(Type an exact answer, using \( \pi \) as needed.)

35. Decide which integral of the Divergence Theorem to use and compute the outward flux of the vector field \( \mathbf{F} = \langle 3x \sin y, - \cos y, 2z \sin y \rangle \) across the surface \( S \), where \( S \) is the boundary of the region bounded by the planes \( x = 2, y = \pi/2, z = 0, \) and \( z = x \).

The outward flux across the surface is \( \square \).
(Type an exact answer, using \( \pi \) as needed.)

36. Fourier's Law of heat transfer (or heat conduction) states that the heat flow vector \( \mathbf{F} \) at a point is proportional to the negative gradient of the temperature; that is, \( \mathbf{F} = -k \nabla T \), which means that heat energy flows from hot regions to cold regions. The constant \( k \) is called the conductivity, which has metric units of \( \text{J/m-s-K} \) or \( \text{W/m-K} \). A temperature function \( T \) for a region \( D \) is given below. Find the net outward heat flux

\[ \iiint_D \mathbf{F} \cdot \mathbf{n} \, dS = -k \iint_S \nabla T \cdot \mathbf{n} \, dS \] across the boundary \( S \) of \( D \). It may be easier to use the Divergence Theorem and evaluate a triple integral. Assume that \( k = 1 \).

\[ T(x,y,z) = 150 + e^{-z}; \quad D = \{ (x,y,z): 0 \leq x \leq 4, 0 \leq y \leq 1, 0 \leq z \leq 3 \} \]

The net outward heat flux across the boundary is \( \square \).
(Type an exact heat flux, using \( \pi \) as needed.)

37. Find the potential function \( f \) for the field \( \mathbf{F} = 9x \mathbf{i} + 4y \mathbf{j} + 7z \mathbf{k} \).

The potential function \( f \) for the field \( \mathbf{F} = 9x \mathbf{i} + 4y \mathbf{j} + 7z \mathbf{k} \) is \( f(x,y,z) = \square \).
(Use \( C \) as the arbitrary constant.)
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<td>1.</td>
<td>B</td>
</tr>
<tr>
<td>2.</td>
<td>B</td>
</tr>
<tr>
<td>3.</td>
<td>D</td>
</tr>
<tr>
<td>4.</td>
<td>A</td>
</tr>
<tr>
<td></td>
<td>C</td>
</tr>
<tr>
<td>5.</td>
<td>A</td>
</tr>
<tr>
<td></td>
<td>B</td>
</tr>
<tr>
<td></td>
<td>C</td>
</tr>
<tr>
<td></td>
<td>Spheres</td>
</tr>
<tr>
<td></td>
<td>D</td>
</tr>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>6.</td>
<td>\frac{1}{2}</td>
</tr>
<tr>
<td>7.</td>
<td>\sqrt{65}</td>
</tr>
<tr>
<td>8.</td>
<td>54</td>
</tr>
<tr>
<td>9.</td>
<td>negative</td>
</tr>
<tr>
<td></td>
<td>positive</td>
</tr>
<tr>
<td></td>
<td>zero</td>
</tr>
<tr>
<td>10.</td>
<td>\frac{1}{2}</td>
</tr>
<tr>
<td></td>
<td>\pi</td>
</tr>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>No</td>
</tr>
<tr>
<td>11.</td>
<td>A</td>
</tr>
<tr>
<td>12.</td>
<td>B</td>
</tr>
</tbody>
</table>
13. \( A, 2x^2 + \frac{5}{2}y^2 + C \)

14. \( A, 4xy + xz + 4yz + C \)

15. \( 2 \\
\quad 2 \)

16. \( -\frac{3}{2} \)

17. \( b = c \\
\quad c = -2b \)

18. \( -8 \\
\quad -16 \\
\quad \text{No} \)

19. \( B \\
\quad C \\
\quad \frac{3}{4}x^4 - \frac{9}{2}x^2y^2 + \frac{3}{4}y^4 \\
\quad -3xy^3 + 3yx^3 \\
\quad 9x^2 - 9y^2 \\
\quad 9y^2 - 9x^2 \\
\quad 18xy \\
\quad -18xy \)

20. \( 7 \)

21. \( 0 \\
\quad 0 \\
\quad 3y \)

22. \( \text{B} \)

23. \( \text{D} \)

24. \( 4\sqrt{14} \)
25. \(\frac{135}{8}\sqrt{3}\)

26. 0

27. 
\(\begin{align*}
&x \\
&xy - \text{plane} \\
&C \\
&3.5631 \\
&4\pi
\end{align*}\)

28. C

29. 0

30. 0

31. 
\(\begin{align*}
&0 \\
&-2z \\
&0 \\
&A \\
&C
\end{align*}\)

32. C

33. \(96\pi\)

34. 0

35. 12

36. \(4e^{-3} - 4\)

37. \(\frac{9}{2}x^2 + 2y^2 + \frac{7}{2}z^2 + C\)