1. Find two vectors parallel to \( \mathbf{v} \) with three times the magnitude of \( \mathbf{v} \).

\[ \mathbf{v} = \langle 4, 5 \rangle \]

Select the correct two vectors below.

- [ ] A. \[ \left( \frac{4}{\sqrt{41}}, \frac{5}{\sqrt{41}} \right) \]
- [ ] B. \(-15, -12\)
- [ ] C. \(-4, -5\)
- [ ] D. \(12, 15\)
- [ ] E. \(-12, -15\)
- [ ] F. \[ \left( -\frac{4}{\sqrt{41}}, -\frac{5}{\sqrt{41}} \right) \]

2. Define the points \( Q(4, -4) \) and \( R(-3, -28) \). Carry out the following calculation.

Find the unit vector with the same direction as \( \overrightarrow{QR} \).

- [ ]

3. A boat is towed with a force of 162 lb with a rope that makes an angle of 45° to the horizontal. Find the horizontal and vertical components of the force.

- [ ]

(Type exact answers, using radicals as needed.)

4. Let \( \mathbf{u} = \langle 4, 5, -6 \rangle \) and \( \mathbf{v} = \langle 3, -5, 2 \rangle \). Evaluate \( \mathbf{u} + \mathbf{v} \) and \( 3\mathbf{u} - \mathbf{v} \).
5. Which point is farther from the origin, \((-5,4,-2)\) or \((0,0,-7)\)?
Choose the correct answer below.

☐ A. The point \((0,0,-7)\) is farther from the origin.
☐ B. The point \((-5,4,-2)\) is farther from the origin.
☐ C. Both points are the same distance from the origin.

6. Find the coordinates of the vertices A, B, and C of the rectangular box shown to the right.

\[ A = (\text{coordinates}) \]  
(Simplify your answers.)

\[ B = (\text{coordinates}) \]  
(Simplify your answers.)

\[ C = (\text{coordinates}) \]  
(Simplify your answers.)

7. Plot the point whose coordinates are \((2, 0, 5)\).
Choose the correct answer below.

☐ A.  
☐ B.  
☐ C.  
☐ D.
8. Find an equation or inequality that describes the following object.

A ball with center \((-6, 6, 5)\) and radius 1.

Choose the correct answer below.

- **A.** \((x + 6)^2 + (y - 6)^2 + (z - 5)^2 \leq 1\)
- **B.** \((x - 6)^2 + (y + 6)^2 + (z + 5)^2 \geq 1\)
- **C.** \((x + 6)^2 + (y - 6)^2 + (z - 5)^2 = 1\)
- **D.** \((x - 6)^2 + (y + 6)^2 + (z + 5)^2 \leq 1\)

9. Give a geometric description of the following set of points.

\[x^2 + y^2 + z^2 - 4x + 8y - 16z + 35 = 0\]

Select the correct choice below and fill in the answer boxes within your choice.

*(Simplify your answers.)*

- **A.** A ball centered at \((\underline{\underline{}}, \underline{\underline{}}, \underline{\underline{}})\) with radius \(\underline{\underline{}}\)
- **B.** A sphere centered at \(\underline{\underline{}}\) with radius \(\underline{\underline{}}\)
- **C.** The exterior of a ball centered at \(\underline{\underline{}}\) with radius \(\underline{\underline{}}\)

10. Find the approximate angle between the vectors \(\mathbf{a} = \langle 6, -6, 5 \rangle\) and \(\mathbf{b} = \langle -5, 5, 3 \rangle\).

The angle between the vectors \(\mathbf{a}\) and \(\mathbf{b}\) is \(\underline{\underline{}}\)°.

*(Type your answer in degrees. Do not round until the final answer. Then round to the nearest hundredth as needed.)*

11. Find \(\text{proj}_\mathbf{u} \mathbf{v}\) and \(\text{scal}_\mathbf{u} \mathbf{v}\) by inspection without using formulas.

\[\text{proj}_\mathbf{u} \mathbf{v} = \underline{\underline{}}\]

*(Type your answer in terms of \(i\) and \(j\).)*

\[\text{scal}_\mathbf{u} \mathbf{v} = \underline{\underline{}}\]
12. A suitcase is pulled 29 feet along a flat sidewalk with a constant upward force of 80 lb at an angle of 23° with the horizontal. Calculate the work done.

The work done is \[ \square \] foot-pounds.

(Round to the nearest foot-pound as needed.)

13. Let \( \mathbf{u} = \langle u_1, u_2, u_3 \rangle \), and \( \mathbf{v} = \langle v_1, v_2, v_3 \rangle \). Prove the following vector property.

\[ \mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u} \]

Determine \( \mathbf{u} \cdot \mathbf{v} \). Choose the correct answer below.

- \( \square \)A. \( \mathbf{u} \cdot \mathbf{v} = v_1 u_1 + v_2 u_2 + v_3 u_3 \)
- \( \square \)B. \( \mathbf{u} \cdot \mathbf{v} = u_1 v_1 + u_2 v_2 + u_3 v_3 \)
- \( \square \)C. \( \mathbf{u} \cdot \mathbf{v} = \sqrt{u_1^2 + u_2^2 + u_3^2} \cdot \sqrt{v_1^2 + v_2^2 + v_3^2} \)

Determine \( \mathbf{v} \cdot \mathbf{u} \). Choose the correct answer below.

- \( \square \)A. \( \mathbf{v} \cdot \mathbf{u} = v_1 u_1 + v_2 u_2 + v_3 u_3 \)
- \( \square \)B. \( \mathbf{v} \cdot \mathbf{u} = u_1 v_1 + u_2 v_2 + u_3 v_3 \)
- \( \square \)C. \( \mathbf{v} \cdot \mathbf{u} = \sqrt{v_1^2 + v_2^2 + v_3^2} \cdot \sqrt{u_1^2 + u_2^2 + u_3^2} \)

Is the equation \( \mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u} \) true?

- \( \square \) No
- \( \square \) Yes

14. Two of the adjacent sides of parallelogram \( P \) are \( \mathbf{u} = \langle -1, -1, 1 \rangle \) and \( \mathbf{v} = \langle 0, 1, -1 \rangle \). Find the area of \( P \).

The area is \( \square \).
15. Consider the parallelepiped determined by the position vectors $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ (see figure). What is the volume of the parallelepiped?

Choose the correct answer below.

- **A.** $V = |\mathbf{u} \times \mathbf{w}|$
- **B.** $V = |\mathbf{v} \cdot (\mathbf{u} \times \mathbf{w})|$
- **C.** $V = |\mathbf{u} \times \mathbf{v}|$
- **D.** $V = |\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})|$
- **E.** $V = 2|\mathbf{u} \times \mathbf{v}|$
- **F.** $V = 2|\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})|$

16. Find parametric equations for the line through $(2, -5, -9)$ parallel to the $z$-axis.

Choose the correct parameterization.

- **A.** $x = 2$, $y = -5$, $z = -9 + t$, $-\infty < t < \infty$
- **B.** $x = 2$, $y = -5$, $z = -9t + 1$, $-\infty < t < \infty$
- **C.** $x = 2t$, $y = -5t$, $z = -9 + t$, $-\infty < t < \infty$
- **D.** $x = 2t$, $y = -5t$, $z = -9t + 1$, $-\infty < t < \infty$

17. Find an equation of the line. Make a sketch of the line.

The line through $(0,0,0)$ and $(7,6,4)$

Write the equation in parameterized form.

$x = \_\_\_\_$, $y = \_\_\_\_$, $z = \_\_\_\_\_$

(Simplify your answers. Type an expression using $t$ as the variable.)

Choose the correct graph below, where the dashed parts of the line are below the $xy$-plane.

- **A.**
- **B.**
- **C.**
18. Find an equation of the line segment joining the given pair of points.

\((-8, -1, 3)\) and \((-1, 2, -1)\)

Choose the correct answer below.

- **A.** \(\langle x,y,z \rangle = t \langle 7,3,-4 \rangle\), for \(-\infty < t < \infty\)
- **B.** \(\langle x,y,z \rangle = \langle -1,2,-1 \rangle + t \langle 7,3,-4 \rangle\), for \(0 \leq t \leq 1\)
- **C.** \(\langle x,y,z \rangle = \langle -8,-1,3 \rangle + t \langle 7,3,-4 \rangle\), for \(0 \leq t \leq 1\)
- **D.** \(\langle x,y,z \rangle = \langle -8,-1,3 \rangle + t \langle -1,2,-1 \rangle\), for \(-1 < t < 1\)
- **E.** \(\langle x,y,z \rangle = \langle -1,2,-1 \rangle + t \langle 7,3,-4 \rangle\), for \(-\infty < t < \infty\)
- **F.** \(\langle x,y,z \rangle = t \langle 7,3,-4 \rangle\), for \(0 \leq t \leq 1\)

19. Give two pieces of information which, taken together, uniquely determine a plane.

Choose the correct answer below.

- **A.** A normal vector
- **B.** Two noncollinear points
- **C.** Any three points
- **D.** One point and a normal vector

20. Where does the plane \(-2x - 4y + 2z = -8\) intersect the coordinate axes?

The plane intersects the x-axis at \(\square\, \square\, \square\).

(Simplify your answer. Type an ordered triple.)

The plane intersects the y-axis at \(\square\, \square\, \square\).

(Simplify your answer. Type an ordered triple.)

The plane intersects the z-axis at \(\square\, \square\).

(Simplify your answer. Type an ordered triple.)
21. Find an equation of the plane that passes through the point \( P_0(-3, -2, 0) \) with a normal vector \( \mathbf{n} = (1, 4, 2) \). Which of the following equations is an equation of the plane that passes through the point \( P_0(-3, -2, 0) \) with a normal vector \( \mathbf{n} = (1, 4, 2) \)?

- [ ] A. An equation for the plane is \( x - 2y = -14 \).
- [ ] B. An equation for the plane is \( -3x - 2y = -11 \).
- [ ] C. An equation for the plane is \( x + 4y + 2z = -11 \).
- [ ] D. An equation for the plane is \( x + 4y = 16 \).

22. Find the equation for the plane through the points \( P_0(3, -3, 3) \), \( Q_0(0, -1, -3) \), and \( R_0(1, -4, 0) \).

The equation of the plane is [ ].
23. For the following set of planes, determine which pairs of planes in the set are parallel, orthogonal, or identical.

Q: \( x + y - z = 0 \); R: \( y + z = 0 \); S: \( x - y = 0 \); T: \( x + y + z = 0 \)

<table>
<thead>
<tr>
<th>Pair of Planes</th>
<th>Relationship</th>
</tr>
</thead>
<tbody>
<tr>
<td>The planes Q and R are</td>
<td>identical.</td>
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<tr>
<td></td>
<td>neither parallel, orthogonal, nor identical.</td>
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<tr>
<td></td>
<td>orthogonal.</td>
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<tr>
<td></td>
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<tr>
<td></td>
<td>identical.</td>
</tr>
<tr>
<td></td>
<td>parallel.</td>
</tr>
</tbody>
</table>
The planes S and T are \text{neither parallel, orthogonal, nor identical.}

24. Find an equation of the line where the planes Q and R intersect.

\[ Q: -2x + y - z = 1; \quad R: x + y + z = -1 \]

Choose the correct answer below.

\begin{itemize}
  \item [\textbf{A.}] An equation of the line is given by $x = -3 + t$, $y = -2t$, $z = -\frac{3}{2} - t$, where $-\infty < t < \infty$.
  \item [\textbf{B.}] An equation of the line is given by $x = -\frac{3}{2} + 2t$, $y = -3 + t$, $z = -2t$, where $-\infty < t < \infty$.
  \item [\textbf{C.}] An equation of the line is given by $x = -\frac{2}{3} + 2t$, $y = -\frac{1}{3} + t$, $z = -3t$, where $-\infty < t < \infty$.
  \item [\textbf{D.}] An equation of the line is given by $x = -3t$, $y = -\frac{1}{3} + 2t$, $z = -\frac{2}{3} + t$, where $-\infty < t < \infty$.
\end{itemize}
25. Consider the following cylinder in $\mathbb{R}^3$ and complete parts a. and b. below.

$$y - 3x^3 = 0$$

a. Identify the coordinate axis to which the cylinder is parallel.

The cylinder is parallel to the

- [x-axis]
- [y-axis]
- [z-axis]

b. Sketch the cylinder. Choose the correct graph below.

![Graph A](image1)

![Graph B](image2)

![Graph C](image3)

26. Determine the graph of the ellipsoid.

$$x^2 + \frac{y^2}{9} + \frac{z^2}{9} = 1$$

Choose the correct graph of the equation.

![Graph A](image4)

![Graph B](image5)

![Graph C](image6)
27. Determine the graph of the paraboloid.

\[ z = x^2 + 81y^2 \]

Choose the correct graph of the equation.

- **A.**  
  ![Graph A](image)

- **B.**  
  ![Graph B](image)

- **C.**  
  ![Graph C](image)

28. Determine the graph of the hyperbolic paraboloid.

\[ \frac{y^2}{81} - \frac{x^2}{81} = z \]

Choose the correct graph of the hyperbolic paraboloid.

- **A.**  
  ![Graph A](image)

- **B.**  
  ![Graph B](image)

- **C.**  
  ![Graph C](image)
29. Determine whether the following statements are true and give an explanation or counterexample.

a. The plane passing through the point \((1,1,1)\) with a normal vector \(\mathbf{n} = \langle 1,2, -3 \rangle\) is the same as the plane passing through the point \((3,0,1)\) with a normal \(\mathbf{n} = \langle -2, -4, 6 \rangle\). Choose the correct answer below.

- True
- False

b. The equations \(x + y - z = 1\) and \(-x - y + z = 1\) describe the same plane. Choose the correct answer below.

- True
- False

c. Given a plane \(Q\), there is exactly one plane orthogonal to \(Q\). Choose the correct answer below.

- False
- True

d. Given a line \(L\) and a point \(P_0\) not on \(L\), there is exactly one plane that contains \(L\) and passes through \(P_0\). Choose the correct answer below.

- True
- False

e. Given a plane \(R\) and a point \(P_0\), there is exactly one plane that is orthogonal to \(R\) and passes through \(P_0\). Choose the correct answer below.

- False
- True

f. Any two distinct lines in \(\mathbb{R}^3\) determine a unique plane. Choose the correct answer below.

- False
- True

g. If plane \(Q\) is orthogonal to plane \(R\) and plane \(R\) is orthogonal to plane \(S\), then plane \(Q\) is orthogonal to plane \(S\). Choose the correct answer below.
29. (cont.)
○ False
○ True

30. Identify the surface defined by the following equation.

\[2x + z^2 - y^2 = 0\]

The surface defined by the equation is

- an elliptic paraboloid.
- a hyperboloid of one sheet.
- a cylinder.
- a plane.
- a hyperbolic paraboloid.
- an ellipsoid.
- a hyperboloid of two sheets.
- an elliptic cone.

31. Find the angle between the planes \(5x + 8y = 18\) and \(3x + 5y + 9z = 4\).

The radian measure of the acute angle is \(\theta = \square\).

(Type an integer or decimal rounded to the nearest thousandth as needed.)
1. D, E

2. \[ \frac{-7}{25} \]
   \[ \frac{24}{25} \]

3. \[ \frac{81\sqrt{2}}{81\sqrt{2}} \]

4. 
   7
   0
   -4
   9
   20
   -20

5. A

6. 
   4
   -6
   7
   0
   -6
   0
   0
   0
   -6
   7

7. A

8. A

9. B, 2, -4, 8, 7

10. 126.50
11. $$\frac{4i}{4}$$

12. 2136

13. B
   A
   Yes

14. $$\sqrt{2}$$

15. D

16. A

17. 7t
   6t
   4t
   C

18. C

19. D

20. 4
    0
    0
    0
    2
    0
    0
    0
    0
    −4

21. C

22. $$-12x + 3y + 7z = -24$$
<p>| | |</p>
<table>
<thead>
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</thead>
</table>
| 23. | orthogonal.  
    | orthogonal.  
    | neither parallel, orthogonal, nor identical.  
    | neither parallel, orthogonal, nor identical.  
    | neither parallel, orthogonal, nor identical.  
    | orthogonal.  |
| 24. | C |
| 25. | z-axis.  
    | A |
| 26. | C |
| 27. | C |
| 28. | A |
| 29. | True  
    | False  
    | False  
    | True  
    | False  
    | False  
    | False  |
| 30. | a hyperbolic paraboloid. |
| 31. | 0.996 |