1. Match the given function with the appropriate graph.

   \[ \mathbf{r}(t) = (t, -t, t) \]

   Choose the correct graph below.

   ![Graph Options A, B, C, D]

2. Evaluate the following limit.

   \[ \lim_{t \to \infty} \left( 3e^{-t} \mathbf{i} - \frac{2t}{t+1} \mathbf{j} + 2 \tan^{-1}t \mathbf{k} \right) \]

3. Find the domain of the following vector-valued function.

   \[ \mathbf{r}(t) = \sqrt{9-t^2} \mathbf{i} + \sqrt{t} \mathbf{j} + \frac{1}{\sqrt{8+t}} \mathbf{k} \]

4. Differentiate the following function.

   \[ \mathbf{r}(t) = \langle 3t e^{-t}, 4t \ln t, t \sin t \rangle \]
5. Let $\mathbf{v}(t) = \langle t^2, -8t, 7 \rangle$. Compute the derivative of the following function.

$\mathbf{v}(e^t)$

6. Suppose $\mathbf{u}$ and $\mathbf{v}$ are differentiable functions at $t = \pi$ with $\mathbf{u}(\pi) = \langle 0, -4, 3 \rangle$, $\mathbf{u}'(\pi) = \langle 0, -3, 4 \rangle$, $\mathbf{v}(\pi) = \langle 0, -4, -3 \rangle$, and $\mathbf{v}'(\pi) = \langle -2,5, -4 \rangle$. Use this information to evaluate the expressions in parts a. through c. below.

a. $\frac{d}{dt} (\mathbf{u} \cdot \mathbf{v}) \bigg|_{t=\pi} = \square$

b. $\frac{d}{dt} (\mathbf{u} \times \mathbf{v}) \bigg|_{t=\pi} = \langle \square, \square, \square \rangle$

c. $\frac{d}{dt} (\mathbf{u}(t) \cos t) \bigg|_{t=\pi} = \langle \square, \square, \square \rangle$

7. For the following curve, find a tangent vector at the given value of $t$.

$$\mathbf{r}(t) = \left(4t^4,12t^{3/2}, \frac{15}{t}\right), t = 1$$

8. Find the function $\mathbf{r}$ that satisfies the following conditions.

$$\mathbf{r}'(t) = \langle 3t^2,2t,4t^3 \rangle; \mathbf{r}(1) = \langle -5,3,7 \rangle$$
9. Complete parts (a) through (c) below.

a. If \( \mathbf{r}(t) = \langle at, bt, ct \rangle \) with \( \langle a, b, c \rangle \neq \langle 0, 0, 0 \rangle \), show that the angle between \( \mathbf{r} \) and \( \mathbf{r}' \) is constant for all \( t \).

Find \( \mathbf{r}' \).

\[
\mathbf{r}(t) = \langle at, bt, ct \rangle \\
\mathbf{r}'(t) = \langle \boxed{\text{ altruistic}} \rangle
\]

Choose the correct statement based on \( \mathbf{r} \) and \( \mathbf{r}' \).

○ A. Since \( \mathbf{r}(t) \) is linear and goes through the origin, \( \mathbf{r} \) and \( \mathbf{r}' \) are always pointing in the same direction.

○ B. Since \( \mathbf{r}(t) \) is non-linear and goes through the origin, \( \mathbf{r} \) and \( \mathbf{r}' \) are always pointing in the same direction.

○ C. Since \( \mathbf{r}(t) \) is linear and does not go through the origin, \( \mathbf{r} \) and \( \mathbf{r}' \) are always pointing in the same direction.

○ D. Since \( \mathbf{r}(t) \) is non-linear and does not go through the origin, \( \mathbf{r} \) and \( \mathbf{r}' \) are always pointing in the same direction.

Therefore, the angle between \( \mathbf{r} \) and \( \mathbf{r}' \) is constant for all \( t \).

b. If \( \mathbf{r}(t) = \langle x_0 + at, y_0 + bt, z_0 + ct \rangle \), where \( x_0, y_0, \) and \( z_0 \) are not all zero, show that the angle between \( \mathbf{r} \) and \( \mathbf{r}' \) varies with \( t \).

Find \( \mathbf{r}' \).

\[
\mathbf{r}(t) = \langle x_0 + at, y_0 + bt, z_0 + ct \rangle \\
\mathbf{r}'(t) = \langle \boxed{\text{ altruistic}} \rangle
\]

Choose the correct statement based on \( \mathbf{r} \) and \( \mathbf{r}' \).

○ A. Since \( \mathbf{r}(t) \) goes through the point \( (x_0, y_0, z_0) \) and \( \mathbf{r}'(t) \) depends on \( t \), the angle between \( \mathbf{r} \) and \( \mathbf{r}' \) varies with \( t \).

○ B. Since both \( \mathbf{r}(t) \) and \( \mathbf{r}'(t) \) go through the point \( (x_0, y_0, z_0) \), but \( \mathbf{r}'(t) \) depends on \( t \), the angle between \( \mathbf{r} \) and \( \mathbf{r}' \) varies with \( t \).

○ C. Since \( \mathbf{r}(t) \) goes through the point \( (x_0, y_0, z_0) \) and \( \mathbf{r}'(t) \) points in a constant direction, the angle between \( \mathbf{r} \) and \( \mathbf{r}' \) varies with \( t \).

○ D. Since both \( \mathbf{r}(t) \) and \( \mathbf{r}'(t) \) go through the point \( (x_0, y_0, z_0) \), but \( \mathbf{r}'(t) \) points in a constant direction, the angle between \( \mathbf{r} \) and \( \mathbf{r}' \) varies with \( t \).

c. Explain the results of part (a) and (b) geometrically.
Let $c = 0$ and $z_0 = 0$. Choose the graph that shows $\mathbf{r}(t) = \langle at, bt, ct \rangle$ and its derivative. Note that since $c = 0$ and $z_0 = 0$, all curves and vectors will be in the xy-plane. The blue graph is $\mathbf{r}$, the red graph is $\mathbf{r}'$, and the purple graph is when $\mathbf{r}$ and $\mathbf{r}'$ are overlapping. Dotted lines represent the vectors at certain values of $t$.

This pair of graphs illustrates that the angle between $\mathbf{r}$ and $\mathbf{r}'$ is constant when $\mathbf{r}(t) = \langle at, bt, ct \rangle$ and varies with $t$ when $\mathbf{r}(t) = \langle x_0 + at, y_0 + bt, z_0 + ct \rangle$. 
10. Given the position function \( \mathbf{r} \) of a moving object, explain how to find the velocity, speed, and acceleration of the object.

Choose the correct answer below.

- **A.** To find the velocity, integrate the position function. To find the speed, add the squares of each component of the position function and take the square root. To find the acceleration, integrate the velocity function.
- **B.** To find the velocity, differentiate the position function. To find the speed, add the squares of each component of the velocity function and take the square root. To find the acceleration, differentiate the velocity function.
- **C.** To find the velocity, integrate the position function. To find the speed, add the squares of each component of the velocity function and take the square root. To find the acceleration, differentiate the position function.
- **D.** To find the velocity, differentiate the position function. To find the speed, add the squares of each component of the acceleration function and take the square root. To find the acceleration, integrate the position function.

11. Given the velocity of an object and its initial position, how do you find the position of the object for \( t \geq 0 \).

Choose the correct answer below.

- **A.** Integrate the initial position and add the result to the velocity function.
- **B.** Integrate the velocity function and use the initial position to find the arbitrary constant \( C \).
- **C.** Add the velocity function to the initial position.
- **D.** Differentiate the velocity function and use the initial position to find any missing constants.

12. Consider the following position function. Find (a) the velocity and the speed of the object and (b) the acceleration of the object.

\[ \mathbf{r}(t) = \langle 8, t^6, e^{-t} \rangle, \text{ for } t \geq 0 \]

(a) \( \mathbf{v}(t) = \langle \boxed{} , \boxed{} , \boxed{} \rangle \)

\[ |\mathbf{v}(t)| = \boxed{} \]

(b) \( \mathbf{a}(t) = \langle \boxed{} , \boxed{} , \boxed{} \rangle \)
13. A projectile is fired over horizontal ground from the origin with an initial speed of 50 m/s. What firing angles will produce a range of 250 m?

14. Find the length of the line given by \( \mathbf{r}(t) = \langle 3t, 10t \rangle \) for \( a \leq t \leq b \).

15. Explain what it means for a curve to be parameterized by its arc length.

Choose the correct answer below.

\( \bigcirc \) A. Let \( \mathbf{r}(t) \) represent the curve. The curvature of the curve is given by the function
\[
s(t) = \int_a^t \frac{\left| \mathbf{T}'(t) \right|}{\left| \mathbf{r}'(t) \right|} \, du,
\]
where \( \mathbf{T}'(t) \) is the unit tangent vector. If \( \left| \mathbf{r}'(t) \right| = 1 \) for all \( t \geq a \), then the parameter \( t \) is the arc length, and the curve is "parameterized by its arc length."

\( \bigcirc \) B. Let \( \mathbf{r}(t) \) represent the curve. The arc length of the curve is given by the function
\[
s(t) = \int_a^t \left| \mathbf{v}(u) \right| \, du,
\]
where \( \left| \mathbf{v} \right| = \left| \mathbf{r}' \right| \). If \( \left| \mathbf{v} \right| = 1 \) for all \( t \geq a \), then the parameter \( t \) is the arc length, and the curve is "parameterized by its arc length."

\( \bigcirc \) C. Let \( \mathbf{r}(t) \) represent the curve. The curvature of the curve is given by the function
\[
\kappa(t) = \frac{\left| \mathbf{T}'(t) \right|}{\left| \mathbf{r}'(t) \right|},
\]
where \( \mathbf{T}'(t) \) is the unit tangent vector. If \( \left| \mathbf{r}'(t) \right| = 1 \) for all \( t \), then the parameter \( t \) is the arc length, and the curve is "parameterized by its arc length."
16. Find the unit tangent vector $\mathbf{T}$ and the curvature $\kappa$ for the following parameterized curve.

$$\mathbf{r}(t) = \langle 2t + 1, 5t - 7, 6t + 12 \rangle$$

$\mathbf{T} = \langle \boxed{\quad} \rangle$

(Type exact answers, using radicals as needed.)

$\kappa = \boxed{\quad}$

(Type exact answers, using radicals as needed.)

17. Prove that the line $\mathbf{r}(t) = \langle x_0 + at, y_0 + bt, z_0 + ct \rangle$ is parameterized by arc length provided $a^2 + b^2 + c^2 = 1$.

Recall that $\mathbf{r}'(t) = \mathbf{v}(t)$.

$$\mathbf{r}(t) = \langle x_0 + at, y_0 + bt, z_0 + ct \rangle$$

$\mathbf{v}(t) = \langle \boxed{\quad} \rangle$

Compute $\mathbf{r}'(t)$.

Now, find the magnitude of $\mathbf{v}$ in terms of $a$, $b$, and $c$.

$$|\mathbf{v}(t)| = \boxed{\quad}$$

Simplify the result.

$$|\mathbf{v}(t)| = \boxed{\quad}$$

(Simplify your answer, including any radicals. Use integers or fractions for any numbers in the expression.)

18. Give the practical formula for computing the principal unit normal vector.

Let $\mathbf{r}$ be a parameterized curve. The practical formula for the principle unit normal vector at a point $P$ on the curve at which $\kappa \neq 0$ is evaluated at the value of $t$ corresponding to $P$.

$$\mathbf{N} = \frac{\mathbf{dT}}{\kappa \frac{ds}{dt}}$$

$\mathbf{N} = \frac{\frac{d\mathbf{T}}{dt}}{\boxed{\quad}}$

$\mathbf{N} = \frac{\frac{ds}{dt}}{\boxed{\quad}}$

$$\mathbf{N} = \frac{\frac{d\mathbf{T}}{dt}}{\boxed{\quad}}$$
19. Find the domain of the following function.

\[ h(x,y) = \sqrt{x - 4y + 3} \]
20. Match functions a through d with their surfaces below.

a. \( f(x,y) = \cos xy \)

b. \( g(x,y) = \ln (x^2 + y^2) \)

c. \( h(x,y) = \frac{1}{x - y} \)

d. \( p(x,y) = \frac{1}{1 + x^2 + y^2} \)

a. Determine the graph of \( f(x,y) = \cos xy \). Choose the correct graph below.

b. Determine the graph of \( g(x,y) = \ln (x^2 + y^2) \). Choose the correct graph below.

c. Determine the graph of \( h(x,y) = \frac{1}{x - y} \). Choose the correct graph below.
20. (cont.)

\[ p(x, y) = \frac{1}{1 + x^2 + y^2} \]

Choose the correct graph below.

21. Match the graph of the function \( z = 2^{-(x^2+y^2)}(x^2+y^2) \) to the system of the level curves.

Which of the following level curves matches the given surface graph?
22. A function is defined by $z = x^2y - xy^2$. Identify the independent and dependent variables.

23. How many axes or how many dimensions are needed to graph the function $z = f(x,y)$? Explain.

Choose the correct answer below.

☐ A. Three axes are needed. One axis is needed for each of the independent variables $x$ and $y$, and one axis is needed for the dependent variable $z$.

☐ B. Three axes are needed. One axis is needed for the ordered pair $(x,y)$, one axis is needed for the dependent variable $z$, and one axis is needed for the function $f$.

☐ C. Four axes are needed. One axis is needed for each of the independent variables $x$ and $y$, one axis is needed for the dependent variable $z$, and one axis is needed for the function $f$.

☐ D. Two axes are needed. One axis is needed for each of the independent variables $x$ and $y$.

24. Find the limit by rewriting the fraction first.

$$\lim_{(x,y) \to (3,5)} \frac{xy - 2y - 9x + 18}{x - 2}$$

25. Evaluate the following limit.

$$\lim_{(x,y) \to (4,e^8)} \ln \sqrt{xy}$$
26. By considering different paths of approach, show that the function has no limit as \((x,y) \to (0,0)\).

\[
f(x, y) = -\frac{x}{\sqrt{x^2 + y^2}}
\]

Find the limit as \((x, y) \to (0,0)\) along the path \(y = x\) for \(x > 0\).

Find the limit as \((x, y) \to (0,0)\) along the path \(y = x\) for \(x < 0\).

Why doesn’t the limit exist?
- **A.** The limit does not exist because \(f(x, y)\) is not continuous.
- **B.** The limit does not exist because \(f(x, y)\) has different limits along two different paths in the domain of \(f\) as \(f(x, y)\) approaches \((0,0)\).
- **C.** The limit does not exist because \(f(x, y)\) has the same limits along two different paths in the domain of \(f\) as \(f(x, y)\) approaches \((0,0)\).
- **D.** The limit does not exist because \(f(x, y)\) is not defined at the point \((0,0)\).

27. What three conditions must be met for a function \(f\) to be continuous at the point \((a,b)\)?

Select all that apply.

- **A.** \(f\) is defined at \((a,b)\)
- **B.** \(\lim_{(x,y) \to (a,b)} f(x,y) \neq f(a,b)\)
- **C.** \((a,b)\) is an interior point
- **D.** The domain of \(f\) is a closed set
- **E.** The domain of \(f\) is an open set
- **F.** \((a,b)\) is a boundary point
- **G.** \(\lim_{(x,y) \to (a,b)} f(x,y) = f(a,b)\)
- **H.** \(\lim_{(x,y) \to (a,b)} f(x,y)\) does not exist
- **I.** \(\lim_{(x,y) \to (a,b)} f(x,y)\) exists
- **J.** \(f\) is not defined at \((a,b)\)
28. Suppose you are standing on the surface \( z = f(x,y) \) at the point \((a,b,f(a,b))\). Interpret the meaning of \( f_x(a,b) \) and \( f_y(a,b) \) in terms of slopes or rates of change.

Choose the correct answer below.

\( \square A \). \( f_x(a,b) \) is the slope of the surface in the direction parallel to the x-axis, and \( f_y(a,b) \) is the slope of the surface in the direction parallel to the y-axis, both taken at \((x,y)\).

\( \square B \). \( f_x(a,b) \) is the slope of the surface in the direction parallel to the x-axis, and \( f_y(a,b) \) is the slope of the surface in the direction parallel to the y-axis, both taken at \((a,b)\).

\( \square C \). \( f_x(a,b) \) is the slope of the surface in the direction parallel to the y-axis, and \( f_y(a,b) \) is the slope of the surface in the direction parallel to the x-axis, both taken at \((x,y)\).

\( \square D \). \( f_x(a,b) \) is the slope of the surface in the direction parallel to the y-axis, and \( f_y(a,b) \) is the slope of the surface in the direction parallel to the x-axis, both taken at \((a,b)\).

29. Find the first partial derivatives of the following function.

\[
f(x,y) = x^3 - 7xy + 7y^2
\]

\( f_x(x,y) = \)

\( f_y(x,y) = \)
30. The productivity of a certain country with the utilization of $x$ units of labor and $y$ units of capital is given approximately by the function $f(x,y) = x^{0.78}y^{0.22}$.

a. Evaluate the partial derivatives $f_x$ and $f_y$.

b. Use part (a) to determine the approximate effect on productivity of increasing capital from 500 to 502 units, while keeping labor fixed at 600 units.

c. What would be the approximate effect of decreasing labor from 600 to 598.5 units, while keeping capital fixed at 500 units?

d. Graph the level curves of the production function in the first quadrant for $f = 1, 2, 3$.

e. If you move along the vertical line $x = 3$, in the positive $y$-direction, how does $f$ change?

\[f_x = \square\]

\[f_y = \square\]

b. If labor is fixed at 600 units and capital is increased from 500 to 502 units, then the change in productivity is \square units.

(Do not round until the final answer. Then round to two decimal places as needed.)

c. If capital is fixed at 500 units and labor is decreased from 600 to 598.5 units, then the change in productivity is \square units.

(Do not round until the final answer. Then round to two decimal places as needed.)

d. Choose the correct graph below. Each graph has the viewing window $[0,5,0.5]$ by $[0,10,1]$.

\[\square A.\] \[\square B.\] \[\square C.\] \[\square D.\]

e. As you move along the vertical line $x = 3$, in the positive $y$-direction, $f$ increases by \square smaller amounts. larger amounts. a constant amount.
31. Assuming that \(3x^3 + 2x^2y - 3y^3 = 0\) defines \(y\) as a differentiable function of \(x\), find \(\frac{dy}{dx}\).

\[
\frac{dy}{dx} = \boxed{} 
\]

32. Use a tree diagram to write the Chain Rule formula for \(\frac{dm}{dw}\), \(m\) is a function of \(v\), where \(v\) is a function of \(r\) and \(s\), each of which is a function of \(w\).

Choose the correct tree diagram below.

\[ 
\text{A.} \quad \frac{\partial m}{\partial r} \frac{\partial r}{\partial w} + \frac{\partial m}{\partial s} \frac{\partial s}{\partial w}  
\text{B.} \quad \frac{\partial m}{\partial r} \frac{\partial r}{\partial w} + \frac{\partial m}{\partial s} \frac{\partial s}{\partial w} 
\text{C.} \quad \frac{\partial m}{\partial r} \frac{\partial r}{\partial v} + \frac{\partial m}{\partial v} \frac{\partial v}{\partial w} 
\text{D.} \quad \frac{\partial m}{\partial r} \frac{\partial r}{\partial v} + \frac{\partial m}{\partial v} \frac{\partial v}{\partial w} + \frac{\partial m}{\partial s} \frac{\partial s}{\partial w} 
\]

What is the Chain Rule formula for \(\frac{dm}{dw}\)?

\[ 
\text{A.} \quad \frac{dm}{dw} = \frac{\partial m}{\partial r} \frac{\partial r}{\partial w} + \frac{\partial m}{\partial s} \frac{\partial s}{\partial w}  
\text{B.} \quad \frac{dm}{dw} = \frac{\partial m}{\partial v} \frac{\partial v}{\partial r} \frac{\partial r}{\partial w} + \frac{\partial m}{\partial v} \frac{\partial v}{\partial s} \frac{\partial s}{\partial w} 
\text{C.} \quad \frac{dm}{dw} = \frac{\partial m}{\partial r} \frac{\partial r}{\partial v} \frac{\partial v}{\partial w} + \frac{\partial m}{\partial s} \frac{\partial s}{\partial v} \frac{\partial v}{\partial w} 
\text{D.} \quad \frac{dm}{dw} = \frac{\partial m}{\partial r} \frac{\partial r}{\partial v} \frac{\partial v}{\partial w} + \frac{\partial m}{\partial v} \frac{\partial v}{\partial s} \frac{\partial s}{\partial w} + \frac{\partial m}{\partial s} \frac{\partial s}{\partial w} 
\]
33. Consider the function \( f(x,y) = 6 - \frac{x^2}{4} - y^2 \), whose graph is a paraboloid (see figure).

**a.** Find the value of the directional derivative at the point \((3,0)\) in the direction \( \langle \cos \theta, \sin \theta \rangle \) where \( \theta = \frac{3\pi}{4} \).

**b.** Sketch the level curve through the given point and indicate the direction of the directional derivative from part (a).

**a.** The directional derivative is \( \square \).

(Type an exact answer, using radicals as needed.)

**b.** Choose the correct sketch below.

[Images of level curves]

34. Interpret the magnitude of the gradient vector at a point.

Choose the correct answer below.

○ **A.** The rate of increase of the function in the direction of steepest ascent at the point

○ **B.** The rate of increase of the function along the level curve at the point

○ **C.** The direction along the level curve at the point

○ **D.** The direction of steepest ascent at the point

35. **a.** Find the linear approximation for the following function at the given point.

**b.** Use part (a) to estimate the given function value.

\[ f(x,y) = -x^2 + 3y^2; \ (4, -1); \ \text{estimate } f(3.9, -1.02) \]

**a.** \( L(x,y) = \square \)

**b.** \( L(3.9, -1.02) = \square \) (Type an integer or a decimal.)
36. Use differentials to approximate the change in \( z \) for the given change in the independent variables.

\[ z = \ln(x^{10}y) \text{ when } (x,y) \text{ changes from } (-4,5) \text{ to } (-3.95,4.98) \]

37. Suppose that in a large group of people a fraction \( 0 \leq r \leq 1 \) of the people have flu. The probability that, in \( n \) random encounters, you will meet at least one person with flu is \( P = f(n,r) = 1 - (1 - r)^n \). Although \( n \) is a positive integer, regard it as a positive real number. Complete parts a. through c. below.

a. Compute \( f_1 \) and \( f_n \).

\[ f_r = \]

\[ f_n = \]

b. How sensitive is the probability \( P \) to the flu rate \( r \)? Suppose you meet \( n = 16 \) people. Approximately how much does the probability \( P \) increase if the flu rate increases from \( r = 0.1 \) to \( r = 0.14 \) (with \( n \) fixed)?

\[ \Delta P \approx \] (Round to three decimal places as needed.)

c. Approximately how much does the probability \( P \) increase if the flu rate increases from \( r = 0.9 \) to \( r = 0.94 \)? Again use \( n = 16 \). Choose the correct answer below.

- \( \text{A. } \Delta P \approx 0.132 \)
- \( \text{B. } \Delta P \approx 0.04 \)
- \( \text{C. } \Delta P \approx 6.4 \times 10^{-17} \)
- \( \text{D. } \Delta P \approx 6.4 \times 10^{-16} \)

38. Find all critical points of the following function.

\[ f(x,y) = \frac{1}{3}x^3 - 5y^3 - 5x + 15y - 8 \]
39. If possible, find the absolute maximum and minimum values of the following function on the set $R$.

$$f(x,y) = 4x^2 - 4y^2 + 4; \ R = \{(x,y): x^2 + y^2 \leq 1\}$$

Find the absolute maximum value on the set. Select the correct choice below and, if necessary, fill in the answer boxes within your choice.

☐ A. The absolute maximum value is ___ and occurs at ___.
   (Type an ordered pair. Use a comma to separate answers as needed.)

☐ B. There is no absolute maximum value.

Find the absolute minimum value on the set. Select the correct choice below and, if necessary, fill in the answer boxes within your choice.

☐ A. The absolute minimum value is ___ and occurs at ___.
   (Type an ordered pair. Use a comma to separate answers as needed.)

☐ B. There is no absolute minimum value.

40. If $f_x(a,b) = f_y(a,b) = 0$, does it follow that $f$ has a local maximum or local minimum at $(a,b)$? Explain.

Choose the correct answer below.

☐ A. No. One (or both) of $f_x$ and $f_y$ must also not exist at $(a,b)$ to be sure that $f$ has a local maximum or local minimum at $(a,b)$.

☐ B. Yes. The tangent plane to $f$ at $(a,b)$ is horizontal. This indicates the presence of a local maximum or a local minimum at $(a,b)$.

☐ C. Yes. The point $(a,b)$ is a critical point and must be a local maximum or local minimum.

☐ D. No. It follows that $(a,b)$ is a critical point of $f$, and $(a,b)$ is a candidate for a local maximum or local minimum.
41. Determine whether the following statements are true and give an explanation or counterexample. Assume that $f$ is differentiable at the points in question. Complete parts a through d below.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>a.</td>
<td>The fact that $f_x(2,2) = f_y(2,2) = 0$ implies that $f$ has a local maximum, local minimum, or saddle point at $(2,2)$. Is this statement true or false?</td>
</tr>
<tr>
<td></td>
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<tr>
<td></td>
<td>True. A critical point must be a local maximum, a local minimum, or a saddle point.</td>
</tr>
<tr>
<td></td>
<td>False. If $D(2,2) = 0$, then the nature of the critical point $(2,2)$ is not known.</td>
</tr>
<tr>
<td>b.</td>
<td>The function $f$ could have a local maximum at $(a,b)$ where $f_y(a,b) \neq 0$. Is this statement true or false?</td>
</tr>
<tr>
<td></td>
<td>False. A local maximum can only occur where $f_x(a,b) = f_y(a,b) = 0$.</td>
</tr>
<tr>
<td></td>
<td>True. A critical point $(a,b)$ occurs where either $f_x(a,b) = 0$ or $f_y(a,b) = 0$.</td>
</tr>
<tr>
<td>c.</td>
<td>The function $f$ could have both an absolute maximum and an absolute minimum at two different points that are not critical points. Is this statement true or false?</td>
</tr>
<tr>
<td></td>
<td>True. The absolute extrema can occur on the boundary of the domain of $f$.</td>
</tr>
<tr>
<td></td>
<td>False. Any extrema, local or absolute, can only occur at a critical point of $f$.</td>
</tr>
<tr>
<td>d.</td>
<td>The tangent plane is horizontal at a point on a surface corresponding to a critical point. Is this statement true or false?</td>
</tr>
<tr>
<td></td>
<td>False. Consider the function $g(x,y) = \frac{1}{xy}$. A critical point exists at $(0,0)$ since neither partial derivative of $g$ exists at $(0,0)$. The tangent plane is vertical.</td>
</tr>
<tr>
<td></td>
<td>True. Since $f_x(a,b) = f_y(a,b) = 0$, there is no instantaneous change in $z = f(a,b)$ at the point $(a,b)$. This implies that the tangent plane is horizontal.</td>
</tr>
</tbody>
</table>
42. Use Lagrange multipliers to find the maximum and minimum values of \( f(x, y) = xy \) (if they exist) subject to the constraint \( x^2 + y^2 - 16 = 0 \).

If there is a maximum value, what is it? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- A. The maximum value is \( \_ \_ \_ \_ \).
- B. There is no maximum value.

If there is a minimum value, what is it? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- A. The minimum value is \( \_ \_ \_ \_ \).
- B. There is no minimum value.

43. The following figure shows the level curves of \( f \) and the constraint curve \( g(x, y) = 0 \). Estimate the maximum and minimum values of \( f \) subject to the constraint.

If there is a maximum value, what is it? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- A. The maximum value is \( \_ \_ \_ \_ \).
- B. There is no maximum value.

If there is a minimum value, what is it? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- A. The minimum value is \( \_ \_ \_ \_ \).
- B. There is no minimum value.

44. A cylindrical can has a volume of \( 1458\pi \) cm\(^3\). What dimensions yield the minimum surface area?

The radius of the can with the minimum surface area is \( \_ \_ \_ \_ \) cm.
(Simplify your answer.)

The height of the can with the minimum surface area is \( \_ \_ \_ \_ \) cm.
(Simplify your answer.)
If \( f(x,y,z) = 2x^2 + 5y^2 + 3z^2 \) and \( g(x,y,z) = 6x + 8y - 9z + 12 = 0 \), write the Lagrange multiple conditions that must be satisfied by a point that maximizes or minimizes \( f \) subject to \( g(x,y,z) = 0 \).

Choose the correct answer below.

- **A.** \( 5x = 6\lambda, 10y = 3\lambda, 8z = 9\lambda, 12x^2 + 3y^2 + 5z^2 = 0 \)
- **B.** \( 8x = -8\lambda, 9y = 6\lambda, 6z = -6\lambda, 8x + 9y - 6z + 12 = 0 \)
- **C.** \( 4x = 6\lambda, 10y = 8\lambda, 6z = -9\lambda, 6x + 8y - 9z + 12 = 0 \)
- **D.** \( 10x = 9\lambda, 4y = -8\lambda, 6z = 6\lambda, 2x^2 + 5y^2 + 3z^2 = 0 \)
1. \( D \)

2. \(-2j + \pi k\)

3. \( D, 0, 3 \)

4. \[
\begin{align*}
3 e^{-t(1-t)} \\
4(1 + \ln t) \\
t \cos t + \sin t
\end{align*}
\]

5. \[
\begin{align*}
2 e^{2t} \\
-8 e^t \\
0
\end{align*}
\]

6. \[
\begin{array}{l}
-32 \\
26 \\
-6 \\
-8 \\
0 \\
3 \\
-4
\end{array}
\]

7. \( A, 16, 18, -15 \)

8. \[
\begin{align*}
t^3 - 6 \\
t^2 + 2 \\
t^4 + 6
\end{align*}
\]

9. \[
\begin{array}{l}
a \\
b \\
c \\
A \\
a \\
b \\
c \\
C \\
A \\
C
\end{array}
\]

10. \( B \)
11. B

12. \[ \begin{align*}
0 \\
6t^5 \\
- e^{-t} \\
\sqrt{36t^{10} + e^{-2t}} \\
0 \\
30t^4 \\
e^{-t}
\end{align*} \]

13. 39.26, 50.74

14. \( \sqrt{109} \) (b – a)

15. B

16. \( \begin{align*}
\frac{2}{\sqrt{65}} \\
\frac{5}{\sqrt{65}} \\
\frac{6}{\sqrt{65}} \\
0
\end{align*} \)

17. \( a \) \\
\( b \) \\
\( c \) \\
\( \sqrt{a^2 + b^2 + c^2} \) \\
1

18. \( \begin{align*}
N = \frac{dT}{dt} \\
\left| \frac{d^2T}{dt^2} \right|
\end{align*} \)

19. D, 4y – 3
<p>| | |</p>
<table>
<thead>
<tr>
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<th></th>
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</thead>
</table>
| 20. | A  
   | A  
   | C  
   | A  |
| 21. | A  |
| 22. | $x, y$  
    | $z$ |
| 23. | A  |
| 24. | A, $-4$ |
| 25. | A, $4 + \ln 2$ |
| 26. | $-\frac{1}{\sqrt{2}}$  
    | $\frac{1}{\sqrt{2}}$  
    | B  |
| 27. | A, G, I |
| 28. | B  |
| 29. | $3x^2 - 7y$  
    | $-7x + 14y$ |
30. \[ \frac{0.78y^{0.22}}{x^{0.22}} \]
\[ = \frac{0.22x^{0.78}}{y^{0.78}} \]
0.51
- 1.12
C
increases
smaller amounts.

31. \[ -\frac{9x^2 + 4xy}{2x^2 - 9y^2} \]

32. D
B

33. \[ \frac{3\sqrt{2}}{4} \]
A

34. A

35. \[ -8x - 6y + 13 \]
- 12.08

36. - 0.129

37. \[ n(1 - r)^{n-1} \]
\[ - (1 - r)^n \ln (1 - r) \]
0.132
D

38. A, \( (\sqrt{5}, 1), (\sqrt{5}, -1), (-\sqrt{5}, 1), (-\sqrt{5}, -1) \)

39. A, 8, \( (1,0), (-1,0) \)
A, 0, \( (0, -1), (0,1) \)

40. D
41. True. A critical point must be a local maximum, a local minimum, or a saddle point. 
   False. A local maximum can only occur where \( f_x(a,b) = f_y(a,b) = 0 \).
   True. The absolute extrema can occur on the boundary of the domain of \( f \).
   True. Since \( f_x(a,b) = f_y(a,b) = 0 \), there is no instantaneous change in \( z = f(a,b) \) at the point \( (a,b) \). This implies that the tangent plane is horizontal.

| 42 | A, 8  
   | A, -8 |

| 43 | A, 6  
   | A, 2  |

| 44 | 9  
   | 18 |

| 45 | C |