For the following function, find the intervals on which $f(x)$ is increasing and the intervals on which $f(x)$ is decreasing. Then find all local minimums and all local maximums.

$$f(x) = x^{\frac{1}{3}}(4 - x)$$  

$$f'(x) = \frac{1}{3}x^{-\frac{2}{3}}(4 - x) + x^{\frac{1}{3}}(-1)$$

First simplify $f'(x)$ by writing it as a single fraction:

$$f'(x) = \frac{4 - x}{3x^{\frac{2}{3}}} - \frac{x^{\frac{1}{3}}}{3x^{\frac{2}{3}}}$$

Test $x = 4$:

$$\frac{(4)(-)}{(4)(+)} = \frac{(4)}{(4)} = +$$

Test $x = \frac{1}{2}$:

$$\frac{(4)(+)}{(\frac{1}{2})(+)} = \frac{(4)}{4} = +$$

Test $x = -1$:

$$\frac{(\frac{1}{2})(-)}{(\frac{1}{2})(-)} = \frac{(\frac{1}{2})}{(\frac{1}{2})} = -$$

$f(x)$ increases anywhere $f'(x)$ is positive and $f(x)$ decreases anywhere $f'(x)$ is negative, so:

Increases: $(0,1)$

Decreases: $(-\infty, 0) \cup (1, \infty)$

Furthermore, based on arrows underneath sign chart, there is a local min @ $x=0$ and a local max @ $x=1$. Plug these back into $f(x)$ to get the complete point:

**Local Min:** $(0, 0)$

**Local Max:** $(1, 3)$
Class Examples
For the following functions, find the interval on which \( f(x) \) is increasing (recall that \( f(x) \) is increasing if \( f'(x) > 0 \)).

a) \( f(x) = -x^3 + 9x \) \hspace{1cm} f'(x) = -3x^2 + 9

b) \( f(x) = x\sqrt{9 - x^2} \) \hspace{1cm} f'(x) = \sqrt{9 - x^2} + x \left( \frac{1}{2} (9 - x^2)^{-\frac{1}{2}} \right) (-2x)

For the following functions, find the interval on which \( f(x) \) is decreasing (recall that \( f(x) \) is increasing if \( f'(x) < 0 \)).

c) \( f(x) = 3x^4 - 16x^3 + 24x^2 \) \hspace{1cm} f'(x) = 12x^3 - 48x^2 + 48x

d) \( f(x) = x^{\frac{3}{2}} (x - 5) \) \hspace{1cm} f'(x) = \frac{2}{3} x^{-\frac{1}{2}} (x - 5) + x^{\frac{1}{2}} (1)

For the following functions, find all the local minimums and local maximums (list your answers as complete ordered pairs)

e) \( f(x) = \frac{1 + x}{\sqrt{x}} \) \hspace{1cm} f'(x) = \frac{(1)\sqrt{x} - (1 + x) \left( \frac{1}{2} x^{-\frac{1}{2}} \right)}{x}
**Homework Problems**

For the following functions, find the intervals on which $f(x)$ is increasing and the intervals on which $f'(x)$ is decreasing.

1) \( f(x) = 3x^4 + 4x^3 - 12x^2 \)  
   \( f'(x) = 12x^3 + 12x^2 - 24x \)

2) \( f(x) = x - 3x^{\frac{2}{3}} \)  
   \( f'(x) = 1 - 2x^{-\frac{1}{3}} \)

For the following functions, find all the local minimums and maximums.

3) \( f(x) = \sqrt{x}(x - 3) \)  
   \( f'(x) = \frac{3}{2} x^{\frac{1}{2}} - \frac{3}{2} x^{-\frac{1}{2}} \)

4) \( f(x) = \frac{1 + x}{\sqrt{x}} \)  
   \( f'(x) = \frac{x^{\frac{1}{2}}(1) - (1 + x)\left(\frac{1}{3} x^{-\frac{1}{2}}\right)}{x^{\frac{3}{2}}} \)