Simplify the expression \( \frac{\sqrt{9 - x^2}}{x^2} \) by making the substitution \( x = 3 \sin \theta \)

Substitute \( x = 3 \sin \theta \)

\[
\frac{\sqrt{9 - (3 \sin \theta)^2}}{(3 \sin \theta)^2}
\]

\[
= \frac{\sqrt{9 - 9 \sin^2 \theta}}{9 \sin^2 \theta}
\]

\[
= \frac{\sqrt{9 (1 - \sin^2 \theta)}}{9 \sin^2 \theta}
\]

\[
= \frac{3 \cos \theta}{9 \sin^2 \theta}
\]

\[
= \frac{\cos \theta}{3 \sin^2 \theta}
\]

\[
= \frac{1}{3} \frac{\cos \theta}{\sin \theta} \cdot \frac{1}{\sin \theta}
\]

\[
= \frac{1}{3} \cot \theta \csc \theta
\]
Algebraically simplify the expression $\sqrt{x^2 - 4x + 13}$ and then make a trig substitution to further simplify the expression.

Need to complete the square under the radical

\[
\sqrt{(x^2 - 4x + \_)+13-\_}
\]

\[
= \sqrt{(x^2 - 4x + 4) + 13 - 4}
\]

\[
= \sqrt{(x-2)^2 + 9}
\]

\[
= \sqrt{(3\tan \theta)^2 + 9}
\]

\[
= \sqrt{9\tan^2 \theta + 9}
\]

\[
= \sqrt{9(\tan^2 \theta + 1)}
\]

\[
= \sqrt{9\sec^2 \theta}
\]

\[
= 3\sec \theta
\]

Use trig identities to re-write $\frac{\sin^3 x}{\cos x}$ as a product.

Split up $\sin^3 \theta$

\[
\sin^2 x \cdot \sin x
\]

\[
\cos x
\]

\[
\sin^2 x \tan x
\]
Class Examples

a) Simplify the expression \(\frac{x}{\sqrt{x^2 + 4}}\) by making the substitution \(x = 2 \tan \theta\)

b) Simplify the expression \(\frac{1}{x^2\sqrt{x^2 - 4}}\) by making the substitution \(x = 2 \sec \theta\)

c) Use trig identities to re-write \(\frac{\cos \theta}{\sin^2 \theta}\) as a product.

d) Use trig identities to simplify \((\sin x + \cos x)^2\) as much as possible.

e) Algebraically simplify the expression \(\sqrt{x^2 + 4x + 5}\) and then make a trig substitution to further simplify the expression.
Homework Problems

1) Simplify the expression \( \frac{1}{\sqrt{x^2 + 10}} \) by making the substitution \( x = 4\tan \theta \)

2) Simplify the expression \( \frac{\sqrt{x^2 - 9}}{x^3} \) by making the substitution \( x = 3\sec \theta \)

3) Use trig identities to re-write \( \frac{\tan \theta}{\sec^2 \theta} \) as a product.

4) Use trig identities to simplify \( \sqrt{ (1 + \sin \theta)^2 + \cos^2 \theta} \) as much as possible.

5) Algebraically simplify the expression \( \frac{1}{\sqrt{x^2 - 6x + 13}} \) and then make a trig substitution to further simplify the expression.

6) For a certain problem, you use the substitution \( x = 3\sec \theta \) and get \( 2\sin(2\theta) + C \) for your answer. Express this answer in terms of \( x \) by re-substituting for \( \theta \).