Math 115 Exam 3 (Chapter 5)

This test is closed book/notes. Show all work clearly and in order, and circle your final answers. There are 7 questions on this test for a total of 60 possible points.

(3 pts) 1) Find the domain of the rational expression \( \frac{x^2 - 4}{x + 3} \)

\[ \text{What makes the bottom equal zero?} \]

\[ x + 3 \neq 0 \]

\[ x \neq -3 \]

(3 pts each) 2) Simplify the following rational expressions:

a) \[ \frac{x^2 + 5x + 6}{x^2 - 9} \]

\[ \frac{(x+3)(x+2)}{(x+3)(x-3)} \]

\[ \frac{x+3}{x-3} \]

b) \[ \frac{x^3 + 4x^2 - x - 4}{x + 1} \]

\[ \frac{x^2(x+4) - 1(x+4)}{(x+1)} \]

\[ \frac{(x^2 - 1)(x+4)}{(x+1)} \]

\[ \frac{(x+4)(x-1)(x+4)}{(x+1)} \]

\[ \frac{(x-1)(x+4)}{x+1} \]
(3 pts each) 3) Perform the following operations:

a) \[
\frac{2x^2 - x - 3}{x^2 - 1} \cdot \frac{x + 4}{2x^2 + 3x - 12}
\]

\[
\frac{(2x-3)(x+1)}{(x+1)(x-1)} \cdot \frac{x+4}{(x+4)(3x-3)}
\]

\[
\frac{1}{x-1}
\]

b) \[
\frac{3x + 6}{x^2 + x - 2} \div \frac{6x + 24}{x^2 - x}
\]

\[
\frac{3x+6}{x^2+x-2} \cdot \frac{x^2-x}{6x+24}
\]

\[
\frac{3(x+2)}{(x+2)(x-1)} \cdot \frac{x}{2(x+4)}
\]

\[
\frac{x}{2(x+4)}
\]

c) \[
\frac{x + 3}{x^2 + 4x - 12} + \frac{3}{x^2 + 4x - 12}
\]

\[
\frac{x+6}{x^2+4x-12}
\]

\[
\frac{(x+6)}{(x+6)(x-2)}
\]

\[
\frac{1}{x-2}
\]

d) \[
\frac{2x - 3}{x^2 + 3x - 10} - \frac{x + 2}{x^2 + 3x - 10}
\]

\[
\frac{2x-3-(x+2)}{x^2+3x-10}
\]

\[
\frac{2x-3-x-2}{x^2+3x-10}
\]

\[
\frac{x-5}{x^2+3x-10}
\]

\[
\frac{(x-5)}{(x+5)(x-2)}
\]
(6 pts each) 4) Perform the following operations:

\[
\begin{align*}
\text{a)} & \quad \frac{3}{x^2 - 16} - \frac{2}{x - 4} \\
& = \frac{3}{(x-4)(x+4)} - \frac{2(x+4)}{(x-4)(x+4)} \\
& = \frac{3 - 2x - 8}{(x-4)(x+4)} \\
& = \frac{-2x - 5}{(x-4)(x+4)} \\
\text{b)} & \quad \frac{x}{x^2 + 2x - 3} + \frac{2}{x^2 + x - 2} \\
& = \frac{x}{(x-1)(x+3)} + \frac{2}{(x+2)(x-1)} \\
& = \frac{x(x+3)}{(x-1)(x+3)(x+3)} + \frac{2(x+2)}{(x-1)(x+2)(x+3)} \\
& = \frac{x^2 + 4x + 6}{(x-1)(x+2)(x+3)}
\end{align*}
\]

(3 pts each) 5) Simplify the following complex fractions:

\[
\begin{align*}
\text{a)} & \quad \frac{x^2 x}{x^3 + y^2} + \frac{1}{\frac{x^2 y^2}{x^2 y^2} - \frac{y}{x^2 y^2}} \\
& = \frac{x^3}{x^2 y^2} + \frac{x^2 y^2}{x^2 y^2} - \frac{y}{x^2 y^2} \\
& = \frac{x^3 y^2}{x^2 y^2} - \frac{y^3}{x^2 y^2} \\
\text{b)} & \quad \frac{2x^{-1} + y^{-2}}{x^{-1} y^{-1}} \\
& = \frac{2x^0 y^{-2}}{x^{-1} y^{-1}} \\
& = \frac{2}{x y^2} + \frac{1}{y^2} \\
& = \frac{1}{x y^2} + \frac{x}{y^2} \\
& = \frac{2y^2}{x y^2} + \frac{x}{x y^2} \\
& = \frac{2y^2 + x}{x y^2}
\end{align*}
\]
(3 pts each) 6) Solve the following equations:

a) \( \frac{x}{x+2} = \frac{3}{x-2} \)

\[
x(x-2) = 3(x+2)
\]
\[
x^2 - 2x = 3x + 6
\]
\[
x^2 - 5x - 6 = 0
\]
\[
(x-6)(x+1) = 0
\]

\[
x = 6
\]
\[
x = -1
\]

b) \( \frac{8x}{3} = \frac{3x}{5} \)

\[
40x = 9x
\]
\[
31x = 0
\]

\[
x = 0
\]

c) \( \frac{2}{x+3} - \frac{5x}{x^2 - 6x - 27} = \frac{6}{x-9} \)

\[
\frac{2(x-9)}{(x-9)(x+3)} - \frac{5x}{(x-9)(x+3)} = \frac{6(x+3)}{(x-9)(x+3)}
\]
\[
2(x-9) - 5x = 6(x+3)
\]
\[
2x - 18 - 5x = 6x + 18
\]
\[
x = -4
\]

-3x - 18 = 6x + 18

\[
-9x = 36
\]
\[
x = -4
\]

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d) \( \frac{3}{x+1} + 2 \cdot \frac{x+1}{x+1} = \frac{5}{x+1} \)

\[
3 + 2(x+1) = 5
\]
\[
3 + 2x + 2 = 5
\]
\[
2x = 0
\]

\[
x = 0
\]
(9 pts) 7) A tourist can bicycle 28 miles in the same time as he can walk 8 miles. If he can ride 10 mph faster than he can walk, how fast does he ride? (be sure to list your unknowns, write an equation, and give your final answer in a complete sentence).

\[ D = R \times T \]

\[ D = \frac{28}{x+10} \quad \text{Bike} \]
\[ D = \frac{8}{x} \quad \text{Walk} \]

\[ T = \frac{D}{R} \]

\[ \text{times are same, so} \]

\[ \frac{28}{x+10} = \frac{8}{x} \]

\[ 28x = 8(x+10) \]

\[ 28x = 8x + 80 \]

\[ 20x = 80 \]

\[ x = 4 \]

Bike rate = \( x + 10 = 14 \)

The tourist rides at 14 mph.